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Nicole Aregger†

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Abstract

Motivated by the potential contribution of China's unilateral peg to asset price inflation in the US before the financial crisis of 2007-2009, this paper studies the effect of alternative exchange rate regimes (flexible versus fixed) on the response of asset prices to economic shocks. I use a two-country general equilibrium model with sticky prices and extend earlier work on this topic by making use of a newer method for analyzing portfolio choice in DSGE models. My findings suggest that asset price responses to shocks differ across regimes. In particular, under a fixed regime, which is operated by the foreign country, responses to shocks in the home country are stronger than under a flexible regime. For home asset prices, however, the amplification of shock responses tends to be small. Applied to the US and China, this implies that, under China's prevailing unilateral peg, the Fed's expansionary monetary policy before the crisis resulted in a slightly but not substantially stronger US asset price inflation relative to the one that would have been observed under a floating USD/CNY exchange rate.

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1 Introduction

The financial crisis of 2007-2009 and the subsequent Great Recession have shown that the financial sector has the potential to cause severe global macroeconomic instability. The shock to one country's financial market – the burst of the bubble in the US subprime mortgage market – not only triggered a substantial contraction in domestic, i.e., US, real economic activity, but was transmitted internationally and led to a global economic downturn.

In general, over the past decades the international transmission of financial shocks has become stronger and systemic fragility has increased. One reason for these developments is financial globalization. Financial markets have become increasingly integrated over the past decades, both domestically and internationally, and gross capital flows across countries have been rising. With the increasing scale of gross foreign asset and liability positions, international spillovers from asset price movements have been enhanced and countries have become more exposed to foreign asset price fluctuations. These fluctuations generate changes in the valuation of existing investment positions and hence, have significant wealth redistribution effects.[1] Movements in international asset prices have therefore gained importance.

Asset price movements have also become more affected by increased cross-border capital flows. In the years before the outburst of the financial crisis, prices of US assets, including stocks, bonds and real estate, rose substantially. Some economists argued that monetary policy had been too expansionary and therefore fuelling the asset price bubble.[2] Caballero and Krishnamurthy (2009) emphasized that this argument, and the argument of shortcomings in supervision and regulation, miss the potential contribution of large and sustained capital inflows into US safe assets to asset price inflation in the US. The excess demand for safe assets created incentives for the financial sector to "construct" more such assets and hence encouraged securitization. Thereby it led to an increase in prices of more risky assets (such as mortgage-backed securities). A similar argument was taken up by Bernanke et al. (2011), who stated that large inflows into US safe assets not only led to a rise in prices of these assets and a decrease in yields, but also to an increase in prices of more risky US assets, such as stocks or real estate. The reason for the latter was that, with low interest rates, US, European and other investors were seeking for higher yields. Therefore, they changed the composition of their portfolio in favor of riskier assets, which pushed up prices.

A large part of the capital inflows into US safe assets came from Asian emerging economies. The decision of East Asian countries, such as China, to invest much of their domestic savings in US safe assets was closely related to their choice of the exchange rate regime and their development strategy, as pointed out by Dooley et al. (2004). China and other East Asian countries have supported economic growth by stimulating exports via a one-sided peg against the US dollar. They undervalue the exchange rate by sizable foreign exchange interventions, capital controls and accumulation of reserves. Starting by the end of the 1990s, this strategy has led to an accommodation of large US current account deficits (mostly through accumulation

[1] See Lane and Milesi-Ferretti (2001, 2005); Lane, Philip R. and Milesi-Ferretti, Gian Maria (2003) for a documentation and analysis of the financial globalization process.

[2] See, e.g., Taylor (2007).

of reserve assets). In 2007, for instance, the US current account registered a deficit of USD 711 billion and China accumulated foreign exchange reserves of USD 462 billion.[3] Assuming that two-thirds of the change in China's foreign reserves were invested in US dollars, China financed more than 40% of the US current account deficit in 2007.[4]

Given the potential contribution of large inflows from China to the rise in US safe and risky asset prices before the financial crisis, this paper investigates the extent to which China's prevailing unilateral peg has played a role in this context.[5] In particular, it assesses if it matters for movements in US asset prices whether China has a fixed instead of a floating exchange rate against the US dollar, and if the peg led to an even stronger US asset price inflation before the outburst of the financial crisis. Furthermore, this paper also analyzes more generally whether nominal exchange rate regimes play a role in the international transmission of shocks to financial markets. The extent to which asset markets, in particular stock and bond prices, respond to economic shocks may differ depending on the prevailing regime. Such analysis provides valuable insight on the extent to which the exchange rate regime contributes to asset price movements, which, in turn, have spillover effects on other countries that hold large gross foreign asset and liability positions.

Comparing the response of asset prices to economic shocks across exchange rate regimes relates to the literature on the transmission of shocks to macroeconomic variables and macroeconomic stability under alternative regimes in general. Already the early Mundell-Fleming literature had extensively examined the properties of alternative regimes and suggested that different exchange rate regimes insulate economies from different types of shocks. One argument why fixed regimes induce higher volatility of economic variables than flexible regimes is the following: When exchange rates cannot adjust and nominal prices are rigid in the short run, a peg implies that the terms of trade cannot move much and a real shock results in price distortions and misallocations of resources. Under flexible exchange rates, in contrast, the nominal exchange rate can adjust in order to mitigate imbalances between two countries, created by country-specific shocks. Looking at the more recent theoretical literature, most of the studies that compare alternative exchange rate regimes are interested in particular qualitative questions (e.g. welfare comparisons).[6]

Findings from empirical studies suggest that alternative exchange rate regimes have been associated with a small or insignificant difference in macroeconomic performance, see Baxter and Stockman (1989), Flood and Rose (1995, 1999) and Rose (2011). However, these results should be interpreted with care since measuring the impact of the exchange rate regime on macroeconomic variables is empirically difficult. Dividing the world into countries that have fixed as opposed to floating regimes is not a trivial task, which is reflected in multiple conflicting

[3]Sources: Bureau of Economic Analysis (US) and State Administration of Foreign Exchange (China).

[4]The exact composition of China's foreign exchange reserves is not publicly available. But in general, it is assumed that about two-thirds of *total* Chinese foreign exchange reserves are held in US dollars.

[5]I abstract from the effect of capital inflows on the wave of securitization.

[6]See, for instance, Bacchetta and van Wincoop (2000), Devereux (1999), Devereux and Engel (2003), Duarte (2003) and Obstfeld and Rogoff (2000). An exception is Collard and Dellas (2002), which provides a comparison of macroeconomic volatility in Germany and France under a unilateral peg (EMS) and a single currency (EMU).

regime classifications.[7] At least part of the weak differences in volatilities of macroeconomic fundamentals, such as growth, trade, inflation and business cycles, across regimes may be explained by using an inappropriate classification of the exchange rate regime as argued in Reinhart and Rogoff (2004). A review of the literature on the classification and performance of alternative exchange rate regimes is provided by Tavlas et al. (2008).

So far, the only existing study that has looked into the topic of asset price responses to shocks under alternative exchange rate regimes is Dellas and Tavlas (2013). Similar to this paper, the authors discussed the role of Chinese investments in US safe assets for asset price inflation in the US. They found that asset price responses to various shocks differ across regimes, but that these differences tend to be small for most shocks. According to their results, the surge in US asset prices before the crisis as a response to various shocks would have been similar under a flexible USD/CNY exchange rate as it actually was the case under China's prevailing unilateral peg regime. For their analysis, Dellas and Tavlas (2013) used a flexible price, two-country portfolio balance model with bonds and equities. A shortcoming of their model is that the portfolio allocation – which is of importance when analyzing asset prices – is ad hoc. Their model abstracts from endogenous portfolio choice for technical simplicity.[8]

What is missing in the literature is a model with endogenous portfolio choice that allows to analyze how the underlying exchange rate regime (flexible versus unilateral peg) affects the transmission of economic shocks on asset prices. The aim of this paper is to fill this gap. I use a simple two-country general equilibrium model and consider the response of equity and bond prices to productivity and monetary shocks. I make use of a newer technique proposed in Devereux and Sutherland (2011) to derive equilibrium portfolios in DSGE models.[9] In general, modelling portfolio choice in a standard DSGE model is not straightforward. Financial assets are distinguishable by their degree of risk and therefore the choice for a certain portfolio allocation depends on second moments where assets' risk characteristics show up. Since the standard approach to solve DSGE models relies on first order approximations around the deterministic steady state, which does not capture second moments, agents are indifferent between different financial assets. The method developed in Devereux and Sutherland (2011) provides a solution to this problem by taking second moments into account.

Apart from making use of a newer technique to allow for endogenous portfolio choice, my model differs in further important aspects from the one in Dellas and Tavlas (2013). First, I abstract from real bond holdings entering agents' utility function to simplify the application of the algorithm described in Devereux and Sutherland (2011). However, under flexible prices, this would automatically result in neutrality of money. Therefore, second, I introduce staggered

[7]The International Monetary Fund (IMF) provided a "de jure" regime classification based on the regime that governments claim to have in place. During the last decade, several "de facto" classifications have been developed. The best-known are Levy-Yeyati and Sturzenegger (2003), Reinhart and Rogoff (2004) and Shambaugh (2004). The mentioned classifications do not overlap well, they vary, e.g., with respect to the number of categories (exchange rate arrangements can seldom be subdivided into fully pegged or fully flexible regimes), or the technique of assigning a country to a category.

[8]In their model, foreign (Chinese) agents can only invest in one financial asset, namely the home currency bond, while home (US) agents have in addition access to home and foreign equities. Asset prices are influenced by the assumption that real bond holdings enter agent's utility function.

[9]A similar technique was proposed by Tille and van Wincoop (2010).

nominal prices à la Calvo in order to restore non-neutrality of money, which is essential when analyzing the role of alternative exchange rate regimes. Third, a more general asset market structure is assumed, where households have symmetric access to financial markets. In this way, I account for the fact that China's financial integration into the world economy has progressed rapidly during the most recent decades and that China has been gradually easing controls on its capital account and foreign exchange markets. This assumption of symmetric access to financial markets has the additional advantage that it simplifies the model. Finally, asset markets are complete, but the model can easily be extended to one with an incomplete asset market structure since the method provided by Devereux and Sutherland (2011) can be implemented with both, complete and incomplete markets. The model should be considered a starting point for further investigation on the response of asset prices to economic shocks under alternative exchange rate regimes.

From my analysis, three main findings emerge. First, and similar to the findings in Dellas and Tavlas (2013), asset price responses to economic shocks differ across exchange rate regimes. More precisely, asset prices in both countries are more responsive to shocks in the home and less responsive to shocks in the foreign country when the foreign country operates a unilateral peg instead of having a floating exchange rate regime. Second, while the differences in the shock responses of home asset prices tend to be small, the differences in the responses of foreign asset prices are more pronounced. Third, the differences are stronger for nominal relative to real shocks. Applied to the US and China, my findings suggest that prices of US bonds and stocks are more responsive to domestic shocks under China's prevailing unilateral peg than under a floating USD/CNY exchange rate, but the extent of this amplification is limited. The reason for this stronger response of US asset prices under the peg is that, in response to a positive US monetary or productivity shock, China relaxes domestic monetary conditions to prevent an appreciation of the Renminbi. Given that prices are sticky in the short run, this slightly amplifies the shock response of US consumption and therefore also of US asset prices. Thus, under the prevailing unilateral peg, the Fed's expansionary monetary policy before the financial crisis resulted in a slightly but not substantially stronger US asset price inflation relative to the one that would have been observed under a floating USD/CNY exchange rate.

The structure of the paper is as follows. Section 2 outlines the model. Section 3 characterizes the model's solution and provides a summary of the solution methods provided by Devereux and Sutherland (2011) to derive equilibrium portfolios. Section 4 derives and explains the optimal portfolio allocation and discusses the response of key macroeconomic variables and asset prices to shocks under both a flexible and a fixed exchange rate regime. Section 5 concludes.

2 Model

The basic structure of the model follows the setup in Devereux and Sutherland (2008), who use a two-country open economy model with imperfect competition, staggered nominal prices à la

Calvo and endogenous portfolio choice.[10] To compare asset prices under alternative exchange rate regimes, elements of Dellas and Tavlas (2013) are incorporated. There is a home (H) and a foreign (F) country, representing the US and China, respectively, each inhabited by households, firms and a monetary authority. Home and foreign households have symmetric preferences and save by holding four types of financial assets, namely home and foreign nominal bonds and home and foreign equities. Home and foreign firms have identical technologies and employ domestic labor to produce a specialized good that can be consumed domestically or exported to the other country. Monetary policy is conducted via a money supply rule. Under a flexible exchange rate regime, this rule is identical for both countries. Under a fixed exchange rate regime, the foreign monetary authority operates a unilateral peg vis-à-vis the home currency, by adjusting the path of foreign money supply.[11][12]

Home and foreign agents face four stochastic shocks, namely country-specific productivity and monetary shocks. This, in combination with agents' access to four assets with independent returns, implies that financial markets are complete. There are enough independent assets (bonds and equities) to allow for perfect risk-sharing across countries.

In what follows the equations relating to the home economy are described. The corresponding equations of the foreign economy are similar and provided in Appendix A.

2.1 Households

The representative domestic household maximizes his expected lifetime utility

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\rho}}{1-\rho} - \frac{L_t^{1+\chi}}{1+\chi} \right) \tag{1}$$

where C_t and L_t denote consumption and hours of labor, respectively. β is the intertemporal discount factor, with $0 < \beta < 1$. $\rho > 0$ denotes the inverse of the intertemporal elasticity of substitution and $\chi > 0$ is the inverse of the elasticity of work effort with respect to the real wage. The household consumes the consumption basket C_t, which is a composite of home and foreign tradable goods consumption $C_{H,t}$ and $C_{F,t}$, respectively. The consumption index is of CES form

$$C_t = \left(\mu^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-\mu)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \tag{2}$$

The parameter $\theta > 0$ measures, from the viewpoint of the domestic consumer, the substitutability between domestic and foreign goods, and $\mu \in (0, 1)$ is the weight of home goods in the

[10] I use the same asset market structure as in their "NBE economy", that is, I allow for trade in home and foreign nominal bonds and equities.

[11] I use the terms "peg" and "fixed exchange rate" interchangeably.

[12] The assumption that foreign money supply is adjusted to maintain a fixed nominal exchange rate parity is made for simplicity. Alternatively, one could adjust the model to allow for non-Ricardian equivalence and then let the foreign government adjusts its holdings of the home currency bond, as done in Dellas and Tavlas (2013) (in their model, non-Ricardian equivalence is a consequence of bonds entering agents' utility function).

consumption basket. I allow for a home bias in tradable goods by assuming $\mu > 0.5$.[13] $C_{H,t}$ is, in turn, an index of consumption of domestic goods given by the CES function

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\phi-1}{\phi}} dj \right)^{\frac{\phi}{\phi-1}} \tag{3}$$

where $j \in [0;1]$ denotes the good variety and $\phi > 1$ is the elasticity of substitution between individual goods. Equivalently, $C_{F,t}$ is an index of consumption of foreign goods. The aggregate consumer price index corresponding to the consumption basket defined above is given by

$$P_t = \left(\mu P_{H,t}^{1-\theta} + (1-\mu)(S_t P_{F,t}^*)^{1-\theta} \right)^{\frac{1}{1-\theta}} \tag{4}$$

$P_{H,t}$ and $P_{F,t}^*$ are the aggregate price indices for home and foreign goods consumption. Foreign variables are denoted with an asterisk '*'. S_t is the nominal exchange rate defined as the price of foreign currency in terms of home currency. Hence, a rise in S_t corresponds to a deprecation of the home currency and an appreciation of the foreign currency.

The representative household faces the following budget constraint

$$\begin{aligned} P_t C_t + e_{H,t} Q_{e,t} + e_{F,t} S_t Q_{e,t}^* + b_{H,t} Q_{b,t} + b_{F,t} S_t Q_{b,t}^* = w_t L_t + e_{H,t-1}(Q_{e,t} + P_t \mathit{\Pi}_t) \\ + e_{F,t-1} S_t (Q_{e,t}^* + P_t^* \mathit{\Pi}_t^*) + b_{H,t-1} R_t Q_{b,t-1} + b_{F,t-1} S_t R_t^* Q_{b,t-1}^* \end{aligned} \tag{5}$$

On the left-hand side we have consumption expenditure and acquisition costs of financial assets. Households have access to four types of financial assets, namely home and foreign equities and a home and foreign nominal bond, where $e_{H,t}, e_{F,t}, b_{H,t}$ and $b_{F,t}$ are the respective asset holdings. While there is a zero net supply of bonds, the net supply of equity is positive and normalized so that in each economy it is equal to one.[14] $Q_{e,t}$, $Q_{e,t}^*$, $Q_{b,t}$ and $Q_{b,t}^*$ are the corresponding nominal asset prices measured in the respective currency. On the right-hand side of equation (5), we have wage income $w_t L_t$, with w_t being the nominal wage. The remaining terms represent the payoffs of financial assets purchased in the previous period. R_t and R_t^* denote the returns on home and foreign nominal bonds, respectively. $\mathit{\Pi}_t$ and $\mathit{\Pi}_t^*$ are the real values of home and foreign firms' profits, which represent the dividend income received from home and foreign firms, respectively.

The domestic budget constraint can be rewritten in a more compact form and in terms of net foreign assets. Let $\alpha_{e_H,t}, \alpha_{e_F,t}, \alpha_{b_H,t}$ and $\alpha_{b_F,t}$ be the real external asset holdings (real external

[13] This assumption is made for two reasons. First, a home bias in consumption is consistent with empirical evidence and second, if instead home and foreign goods are assumed to have equal weight in the consumption basket ($\mu = 0.5$), purchasing power parity (PPP) always holds, and in combination with complete markets, the nominal exchange rate does not respond to productivity shocks (see equation (83)), which is not an interesting case for the analysis of this paper.

[14] This implies that $e_{H,t} + e_{H,t}^* = e_{F,t} + e_{F,t}^* = 1$ and $b_{H,t} + b_{H,t}^* = b_{F,t} + b_{F,t}^* = 0$, where $e_{H,t}^*, e_{F,t}^*, b_{H,t}^*$ and $b_{F,t}^*$ are the asset holdings of the foreign household, measured in terms of the home consumption basket.

expenditure on each asset measured in terms of the home consumption basket) defined as follows

$$\alpha_{e_H,t} \equiv (e_{H,t} - 1)\frac{Q_{e,t}}{P_t} \tag{6}$$

$$\alpha_{e_F,t} \equiv e_{F,t} Q_t \frac{Q^*_{e,t}}{P^*_t} \tag{7}$$

$$\alpha_{b_H,t} \equiv b_{H,t} \frac{Q_{b,t}}{P_t} \tag{8}$$

$$\alpha_{b_F,t} \equiv b_{F,t} Q_t \frac{Q^*_{b,t}}{P^*_t} \tag{9}$$

Q_t is the real exchange rate, defined as the ratio of consumer price indices expressed in the same currency. If $\alpha_{e_H,t} < 0$ ($\alpha_{e_F,t} < 0$), the home household sells real holdings of home equity (foreign equity) to the foreign household. Equivalently, if $\alpha_{b_H,t} < 0$ ($\alpha_{b_F,t} < 0$), the home household sells real holdings of home bonds (foreign bonds) to the foreign household. Positive values imply that trade in financial assets goes in the opposite direction. The sum over all alphas denotes total net claims of the home country on the foreign country in terms of the home consumption basket, i.e., the home country's net foreign assets, $NFA = \alpha_{e_H} + \alpha_{e_F} + \alpha_{b_H} + \alpha_{b_F}$. NFA_t are net foreign assets at the end of period t. Using theses definitions, equation (5) simplifies as follows (see Appendix B for a derivation)

$$P_t C_t + P_t NFA_t = w_t L_t + P_t \Pi_t + P_t \sum_{k \in K} \alpha_{k,t-1} r_{k,t} \tag{10}$$

The final term in this equation is the total return on the home country portfolio, where $\alpha_{k,t-1}$ represents the real external holdings of asset k chosen at the end of period $t-1$ for holding into period t, and $r_{k,t}$ is the real gross return on this asset, realized in period t. $K = \{e_H, e_F, b_H, b_F\}$ is the set of the four assets. Note that, if $\alpha_{e_{H,t-1}} = 0$, then the home household owns 100% of domestic equity at the beginning of period t, earns a zero return on external holdings of home equity and receives $P_t \Pi_t$, which shows up on the right-hand side of equation (10). If α_{e_H} is negative, claims to home profits are transferred to the foreign household via trade in equity shares.

Asset returns are defined to be the payoff of the asset relative to the asset price. Home and foreign equities represent a claim on home and foreign aggregate profits (or dividends), respectively. The real payoff to one unit of home equity purchased in period t at real price $Z_{e,t} = Q_{e,t}/P_t$ is defined to be $\Pi_{t+1} + Z_{e,t+1}$. The equivalent definition applies to the payoff of foreign equity, where the real price of one unit of foreign equity is $Z^*_{e,t} = Q^*_{e,t}/P^*_t$. Thus, the gross real rates of return on a unit of home and foreign equity are

$$r_{e_H,t+1} = \frac{\Pi_{t+1} + Z_{e,t+1}}{Z_{e,t}} \tag{11}$$

$$r_{e_F,t+1} = \frac{\Pi^*_{t+1} + Z^*_{e,t+1}}{Z^*_{e,t}} \frac{Q_{t+1}}{Q_t} \tag{12}$$

The real exchange rate enters equation (12) because asset payoffs are measured in terms of the consumption basket of the respective economy, while asset returns are measured in terms of the home consumption basket.

A home nominal bond purchased in period t at real price $Z_{b,t} = Q_{b,t}/P_t$ represents a claim on one unit of the domestic currency in period $t+1$. The real payoff to this bond is therefore $1/P_{t+1}$. Equivalently, a foreign nominal bond purchased at real price $Z^*_{b,t} = Q^*_{b,t}/P^*_t$ represents a claim on one unit of foreign currency and the real payoff to this bond is $1/P^*_{t+1}$. Hence, the gross real rates of return on the home and foreign bond are

$$r_{b_H,t+1} = \frac{1/P_{t+1}}{Z_{b,t}} \tag{13}$$

$$r_{b_F,t+1} = \frac{1/P^*_{t+1}}{Z^*_{b,t}} \frac{Q_{t+1}}{Q_t} \tag{14}$$

We can further rewrite the domestic budget constraint (10) as a net foreign asset accumulation equation

$$\begin{aligned} P_tC_t + P_tNFA_t = w_tL_t + P_{H,t}Y_t - w_tL_t + P_t \sum_{k\in K} \alpha_{k,t-1} r_{k,t} - P_t \sum_{k\in K} \alpha_{k,t-1} r_{b_F,t} \quad (15) \\ + P_t \sum_{k\in K} \alpha_{k,t-1} r_{b_F,t} \\ \Leftrightarrow P_tC_t + P_tNFA_t = P_{H,t}Y_t + P_t \sum_{k\in K'} \alpha_{k,t-1} r_{xk,t} + P_t \sum_{k\in K} \alpha_{k,t-1} r_{b_F,t} \quad (16) \\ \Leftrightarrow P_tC_t + P_tNFA_t = P_{H,t}Y_t + P_t \sum_{k\in K'} \alpha_{k,t-1} r_{xk,t} + P_tNFA_{t-1} r_{b_F,t} \quad (17) \end{aligned}$$

where the foreign bond is used as a numéraire asset and $r_{xk,t}$ measures the excess return on asset $k \in K'$, with $K' = \{e_H, e_F, b_H\}$. This representation of the budget constraint allows us to solve the portfolio problem for the choice of $\alpha_k \quad \forall k \in K'$, using the algorithm described in Devereux and Sutherland (2011).[15]

Coming back to the household's maximization problem, the representative household chooses consumption, labor, net foreign assets, external holdings of home and foreign equities and external holdings of home bonds. The optimal consumption-leisure tradeoff implies that the marginal rate of substitution between consumption and leisure equals the real wage, i.e.,

$$\frac{L_t^{\chi}}{C_t^{-\rho}} = \frac{w_t}{P_t} \tag{18}$$

The optimal consumption-savings decision and the optimal portfolio choices, that is, the first

[15]Note that under complete asset markets, there are alternative ways to derive the equilibrium portfolio allocation. It is possible to identify an equilibrium macroeconomic allocation independent of financial structure, and then, given this allocation, the implied portfolio which supports the equilibrium can be derived, see, e.g., Engel and Matsumoto (2009) and Kollmann (2006). Under incomplete markets, however, this procedure is not applicable. To easily allow for extensions of my model with incomplete asset markets, the methods provided by Devereux and Sutherland (2011) are used to derive the equilibrium portfolio allocation.

order conditions with respect to NFA_t, $\alpha_{e_H,t}$, $\alpha_{e_F,t}$ and $\alpha_{b_H,t}$ imply

$$C_t^{-\rho} = \beta E_t[C_{t+1}^{-\rho} r_{b_F,t+1}] \tag{19}$$

$$E_t[C_{t+1}^{-\rho}(r_{e_H,t+1} - r_{b_F,t+1})] = 0 \tag{20}$$

$$E_t[C_{t+1}^{-\rho}(r_{e_F,t+1} - r_{b_F,t+1})] = 0 \tag{21}$$

$$E_t[C_{t+1}^{-\rho}(r_{b_H,t+1} - r_{b_F,t+1})] = 0 \tag{22}$$

Equations (19)-(22) can be rewritten in a more familiar form that shows how asset prices are determined. Combining, e.g., equation (20) with equations (19) and (11) yields

$$C_t^{-\rho} = \beta E_t \left[C_{t+1}^{-\rho} r_{e_H,t+1} \right] \tag{23}$$

$$\Leftrightarrow Z_{e,t} = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} (\mathit{\Pi}_{t+1} + Z_{e,t+1}) \right] \tag{24}$$

This is the central asset pricing formula. Given the payoff $\mathit{\Pi}_{t+1} + Z_{e,t+1}$ and given the investor's consumption choices C_t, C_{t+1}, it indicates what market price $Z_{e,t}$ to expect. In other words, asset prices are forward looking and depend on the covariance between future payoffs and the future stochastic discount factor, $\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho}$. The latter represents the home household's intertemporal marginal rate of substitution

$$SDF_{t+1} \equiv \beta \frac{U'(C_{t+1})}{U'(C_t)} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \tag{25}$$

The stochastic discount factor is a measure of the home household's appetite for wealth. The higher it is, the higher the household's appetite for wealth. The prices of the other three assets could be rewritten in a similar way.

2.2 Firms

Firms produce a differentiated product according to technology

$$Y_t(j) = A_t L_t(j) \tag{26}$$

where $j \in [0; 1]$ is the firm-specific or variety index. The home good is produced in a number of varieties or brands over a continuum of a unit mass. Labor is the only factor of production and A denotes productivity. The law of motion of $\log A_t$ is given by the AR(1) process

$$\log A_t = \psi_a \log A_{t-1} + u_t \tag{27}$$

u is a i.i.d. stochastic shock with zero mean and constant variance $\sigma_a^2 > 0$, and ψ_a reflects the persistence of the shock. Prices are sticky and a Calvo-price setting is assumed with firms setting prices in the currency of the country in which they produce. The evolution of the price for home firms' goods is[16]

$$P_{H,t} = [(1-\kappa)\tilde{P}_{H,t}^{1-\phi} + \kappa P_{H,t-1}^{1-\phi}]^{\frac{1}{1-\phi}} \tag{28}$$

$\tilde{P}_{H,t}$ is the price of firms that reset their price in period t. Only a fraction $1-\kappa$ can reset their price in every period. A firm that can reset its price will optimally choose the following newly set price

$$\tilde{P}_{H,t} = \mathcal{M}\frac{E_t \sum_{i=0}^{\infty} \Omega_{t+i}\kappa^i \frac{w_{t+i}}{A_{t+i}} X_{H,t+i}}{E_t \sum_{i=0}^{\infty} \Omega_{t+i}\kappa^i X_{H,t+i}} \tag{29}$$

$X_{H,t}$ is the demand for the home firm's output coming from home and foreign households. $\mathcal{M} \equiv \frac{\phi}{\phi-1}$ is the desired gross markup under flexible prices[17] and Ω_{t+i} is the firm's stochastic nominal discount factor between period t and $t+i$[18]. Aggregate home and foreign country profits are

$$\Pi_t = \frac{P_{H,t}Y_t - w_t L_t}{P_t} \tag{30}$$

$$\Pi_t^* = \frac{P_{F,t}^* Y_t^* - w_t^* L_t^*}{P_t^*} \tag{31}$$

2.3 Additional Elements

As in Dellas and Tavlas (2013), a simple money demand equation of the following form is postulated

$$M_t v = P_t C_t \tag{32}$$

v is the velocity of money, which is the number of times one unit of money is spent to buy goods per time period. I am not incorporating an explicit cash-in-advance (CIA) constraint, but postulate money demand in an ad hoc manner without an explicit justification for why households would want to hold money. If prices were flexible, the only role played by money was to serve as a numéraire, that is, a unit of account, in which prices are stated.[19] Since prices are sticky, however, monetary frictions arise that give money the additional role of affecting the real economy in the short run.

Monetary policy is implemented using a money supply rule as in Dellas and Tavlas (2013), instead of using the interest rate rule in Devereux and Sutherland (2008): The rule of the home monetary authority is standard while the one of the foreign authority is similar to the one in

[16] Henceforth, index j is omitted since home firms are identical and have unit mass.

[17] In the limiting case of no price rigidities ($\kappa = 0$), equation (29) collapses to the optimal price-setting condition under flexible prices: $\tilde{P}_{H,t} = P_{H,t} = \mathcal{M}\frac{w_t}{A_t}$, where $\frac{w_t}{A_t}$ are the marginal costs (in nominal terms) and $\mathcal{M}$ is the desired markup.

[18] If we assume that firms discount future profits at the same discount rate as their shareholders, then Ω_{t+i} is a function of both, the home and foreign intertemporal rates of substitution. Under complete markets, these two rates are identical.

[19] Such an economy is often referred to as a cashless economy, as described by Woodford (2003).

Devereux (2004). That is, under the fixed exchange rate regime (unilateral peg), the foreign monetary authority adjusts the path of foreign money supply such that the nominal exchange rate is constant.

$$\log M_t = \psi_m \log M_{t-1} + m_t \tag{33}$$

$$\log M_t^* = \psi_m \log M_{t-1}^* - \epsilon \log\left(\frac{S_t}{S}\right) + m_t^* \tag{34}$$

m and m^* are i.i.d. stochastic shocks with zero mean and constant variance $\sigma_m^2 > 0$, ψ_m reflects the persistence of the shocks and S is the steady state value of the nominal exchange rate. The parameter ϵ is a measure of the exchange rate flexibility. A higher ϵ can be interpreted as a policy placing more emphasis on exchange rate stability. Under the flexible exchange rate regime, ϵ is set to zero and under the unilateral peg, ϵ approaches infinity.

The terms of trade are given by

$$\tau_t = \frac{P_{F,t}^* S_t}{P_{H,t}} \tag{35}$$

They reflect, from the domestic country's perspective, the relative price of foreign goods imported in terms of home goods exported. An increase in τ_t implies a relative increase in expenditure on imports and hence, a deterioration of the home terms of trade. Since the produced good is a differentiated good in both countries (we have monopolistically competitive firms), we cannot apply the law of one price for home and foreign goods, that is, τ_t is not equal to one.

The real exchange rate is given by

$$Q_t = \frac{P_t^* S_t}{P_t} \tag{36}$$

The assumption of a home bias in consumption ($\mu > 0.5$) implies that purchasing power parity (PPP) does not hold. The home and foreign consumer price indices do not move in step, so movements in the relative price of the foreign good lead to movements in the real exchange rate. In this point, the model slightly differs from the one in Devereux and Sutherland (2008), where μ is set to 0.5 and therefore the home and foreign consumption baskets are identical. In their model, PPP holds and the real exchange rate is always equal to 1.

Domestic goods market clearing implies that

$$Y_t = X_{H,t} \tag{37}$$

$$\Leftrightarrow Y_t = C_{H,t} + C_{H,t}^* \tag{38}$$

$$\Leftrightarrow Y_t = \mu \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t + (1-\mu)\left(\frac{P_{H,t}}{S_t P_t^*}\right)^{-\theta} C_t^* \tag{39}$$

where, $C_{H,t} = \mu \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t$ and $C_{H,t}^* = (1-\mu)\left(\frac{P_{H,t}}{S_t P_t^*}\right)^{-\theta} C_t^*$ are the demand functions for the home good resulting from the home and foreign households' optimal allocation of any given expenditure between home and foreign goods. Equivalently, foreign goods market clearing reads

$$Y_t^* = C_{F,t}^* + C_{F,t} \tag{40}$$

$$\Leftrightarrow Y_t^* = \mu \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\theta} C_t^* + (1-\mu) \left(\frac{P_{F,t}^* S_t}{P_t} \right)^{-\theta} C_t \tag{41}$$

where $C_{F,t} = (1-\mu) \left(\frac{P_{F,t}^* S_t}{P_t} \right)^{-\theta} C_t$ and $C_{F,t}^* = \mu \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\theta} C_t^*$ are the demand functions for the foreign good.

Market clearing in asset markets implies that[20]

$$\alpha_{k,t-1} + \alpha_{k,t-1}^* = 0 \quad \forall k \in K \tag{42}$$

where $\alpha_{k,t-1}^*$ denotes the foreign household's real external holdings of asset k, measured in terms of the home consumption basket.

Note that for the later analysis it is useful to define a measure for the current account position of the home country. Define $\Delta NFA_t \equiv NFA_t - NFA_{t-1}$ to be the change in net foreign assets, then the home country budget constraint may be written as

$$P_t C_t + P_t \Delta NFA_t = P_{H,t} Y_t + P_t \sum_{k \in K'} \alpha_{k,t-1} r_{xk,t} + P_t NFA_{t-1}(r_{b_F,t} - 1) \tag{43}$$

$$\Leftrightarrow \Delta NFA_t = \frac{P_{H,t}}{P_t} Y_t - C_t + NFA_{t-1}(r_{b_F,t} - 1) + \sum_{k \in K'} \alpha_{k,t-1} r_{xk,t} \tag{44}$$

Similar as in Devereux and Sutherland (2009), I define

$$CA \equiv \frac{P_{H,t}}{P_t} Y_t - C_t + NFA_{t-1}(r_{b_F,t} - 1) \tag{45}$$

to be a measure of the current account and the remaining term in equation (44), $\sum_{k \in K'} \alpha_{k,t-1} r_{xk,t}$, to represent the unanticipated valuation effect arising from capital gains and losses on gross external assets and liabilities. Such valuation effects account for an important and increasing part of the dynamics of a country's net foreign asset position as pointed out by Gourinchas and Rey (2014).

3 Model Solution

As in most applications of DSGE models, it is not possible to obtain the exact solution of the model. Therefore, the model is solved by linear approximation around the steady state. It consists of the following 35 equations: (4), (18)-(22), (26)-(29), (32), along with the analogous equations for the foreign country (see Appendix A) and equations (11)-(14), (17), (30), (31),

[20]Remember that there is a positive net supply of home and foreign equity normalized to 1, i.e., $e_H + e_H^* = 1$ and $e_F + e_F^* = 1$. Despite this, the sum over home and foreign external equity holdings is zero. For home equity, e.g., combining $e_H + e_H^* = 1$ with the definitions of α_{e_H} and $\alpha_{e_H}^*$ yields $\alpha_{e_H} + \alpha_{e_H}^* = (e_H - 1)Z_e + e_H^* Z_e = 0$. Equivalently, we can show for foreign equity that the left hand side of equation (42) is zero.

(33)-(36), (39) and (41). The 35 endogenous variables are $P_H, \tilde{P}_H, P, M, Y, A, L, Z_e, Z_b, \mathit{\Pi}, C, w$, their foreign counterparts, and $\tau, S, Q, NFA, \alpha_{e_H}, \alpha_{e_F}, \alpha_{b_H}, r_{e_H}, r_{e_F}, r_{b_H}$ and r_{b_F}. Note that α_{b_F} and the portfolio choice variables of the foreign household, $\alpha_k^* \ \forall k \in K$, do not show up in this list. The reason is that once $NFA, \alpha_{e_H}, \alpha_{e_F}$ and α_{b_H} are determined, these variables are also determined as $\alpha_{b_F} = NFA - \alpha_{e_H} - \alpha_{e_F} - \alpha_{b_H}$ by definition and $\alpha_k^* = -\alpha_k \ \forall k \in K$ from the market clearing conditions. Thus, in what follows, we focus only on α_{e_H}, α_{e_F}, α_{b_H}.

3.1 Steady State

In the non-stochastic steady state four of the above equations, namely the foreign household's optimal consumption-savings decision and portfolio choices (equations (103)-(106) in Appendix A), are redundant since they are identical to the home country's counterparts, equations (19)-(22). Both sets of equations imply the same two things. First, in the steady state all assets yield the same return, $r_{e_H} = r_{e_F} = r_{b_H} = r_{b_F} \equiv r$, and second, the discount rate on savings is equal to the subjective discount rate, $\frac{1}{r} = \beta$. The former implies that all assets are perfect substitutes and that for a given amount of net foreign assets, households are indifferent between any composition of their portfolio. The steady state proves inadequate to capture the different risk characteristics of financial assets and therefore, it does not tie down a unique portfolio allocation. As a consequence, α_{e_H}, α_{e_F} and α_{b_H} are indeterminate in the steady state. To tie down a unique portfolio allocation, i.e., to find a reasonable portfolio allocation around which the model is approximated, the method suggested in Devereux and Sutherland (2011) is applied.[21] Their approximation techniques consider second order moments where assets' risk characteristics show up to derive equilibrium portfolios in DSGE models. A more detailed description of their method is provided in the next subsection. The present model is not sufficiently small to derive an analytical solution but the method allows to find numerical results.

The latter, $\frac{1}{r} = \beta$, implies that in the steady state households are indifferent between any size of their portfolio, i.e., their net foreign asset position. As in Devereux and Sutherland (2008) and Dellas and Tavlas (2013), the model is approximated around the symmetric steady state, which implies that net foreign assets are zero. This allows to derive the solution for all variables in the non-stochastic steady state (except for α_{e_H}, α_{e_F} and α_{b_H}). An overview of these steady state values is provided in Appendix C.

3.2 Approximation Methods Proposed by Devereux and Sutherland (2011)

In this subsection, the approximation methods developed in Devereux and Sutherland (2011) to derive the optimal steady state portfolio allocation are explained. In a few words, their method

[21] Also the first order approximation of the model does not tie down a unique portfolio allocation. The first order approximation of equations (20)-(22) and (104)-(106) (see equations (147)-(149) and (151)-(153) in Appendix D) imply that all expected returns are identical and therefore they prove inadequate to capture the different risk characteristics of financial assets. Any value assigned to α_{e_H}, α_{e_F} and α_{b_H} is consistent with an equilibrium and hence, α_{e_H}, α_{e_F} and α_{b_H} are indeterminate up to their first order approximation. Devereux and Sutherland (2010) provide a solution to find the dynamics of portfolio holdings. However, they are not relevant for the purpose of this paper.

suggests to combine a second order approximation of the portfolio choice equations with a first order approximation of the remaining equations of the model.

Let us start with the second order approximation of the portfolio choice equations. The reason why we need to consider second order approximations is that assets are only distinguishable in terms of their risk characteristics. These risk characteristics only show up in second order moments. The second order approximation of the portfolio choice equations provide a condition which captures the impact of the portfolio choices on the correlation between portfolio returns and relative marginal utilities of consumption and thereby makes it possible to tie down the steady state portfolio allocation.

The optimality conditions for the home and foreign households' choice of real external holdings of home and foreign equity as well as home bonds are given by equations (20)-(22) and (104)-(106), respectively. Taking second order approximations of these six equations yields (for a derivation see Appendix E):[22]

$$E_t[\hat{r}_{xe_H,t+1} - \rho\hat{C}_{t+1}\hat{r}_{xe_H,t+1} + \frac{1}{2}[\hat{r}^2_{e_H,t+1} - \hat{r}^2_{b_F,t+1}]] = 0 \tag{46}$$

$$E_t[\hat{r}_{xe_F,t+1} - \rho\hat{C}_{t+1}\hat{r}_{xe_F,t+1} + \frac{1}{2}[\hat{r}^2_{e_F,t+1} - \hat{r}^2_{b_F,t+1}]] = 0 \tag{47}$$

$$E_t[\hat{r}_{xb_H,t+1} - \rho\hat{C}_{t+1}\hat{r}_{xb_H,t+1} + \frac{1}{2}[\hat{r}^2_{b_H,t+1} - \hat{r}^2_{b_F,t+1}]] = 0 \tag{48}$$

$$E_t[\hat{r}_{xe_H,t+1} - \rho\hat{C}^*_{t+1}\hat{r}_{xe_H,t+1} + \frac{1}{2}[\hat{r}^2_{e_H,t+1} - \hat{r}^2_{b_F,t+1}] - \hat{Q}_{t+1}\hat{r}_{xe_H,t+1}] = 0 \tag{49}$$

$$E_t[\hat{r}_{xe_F,t+1} - \rho\hat{C}^*_{t+1}\hat{r}_{xe_F,t+1} + \frac{1}{2}[\hat{r}^2_{e_F,t+1} - \hat{r}^2_{b_F,t+1}]] - \hat{Q}_{t+1}\hat{r}_{xe_F,t+1}] = 0 \tag{50}$$

$$E_t[\hat{r}_{xb_H,t+1} - \rho\hat{C}^*_{t+1}\hat{r}_{xb_H,t+1} + \frac{1}{2}[\hat{r}^2_{b_H,t+1} - \hat{r}^2_{b_F,t+1}]] - \hat{Q}_{t+1}\hat{r}_{xb_H,t+1}] = 0 \tag{51}$$

where a hat indicates the log deviation of a variable from its value in the non-stochastic steady state. Exceptions are $\hat{r}_{xe_H,t}$, $\hat{r}_{xe_F,t}$ and $\hat{r}_{xb_H,t}$, which are denoted as $\hat{r}_{xe_H,t} = \hat{r}_{e_H,t} - \hat{r}_{b_F,t}$, $\hat{r}_{xe_F,t} = \hat{r}_{e_F,t} - \hat{r}_{b_F,t}$ and $\hat{r}_{xb_H,t} = \hat{r}_{b_H,t} - \hat{r}_{b_F,t}$ (further exceptions are $\hat{CA}_t$ and $N\hat{F}A_t$ which are defined below).[23] Subtracting (46) from (49), (47) from (50), and (48) from (51) leads to the following portfolio orthogonality conditions:

$$E_t\left[(\hat{C}_{t+1} - \hat{C}^*_{t+1} - \hat{Q}_{t+1}/\rho)\hat{r}_{xe_H,t+1}\right] = 0 \tag{52}$$

$$E_t\left[(\hat{C}_{t+1} - \hat{C}^*_{t+1} - \hat{Q}_{t+1}/\rho)\hat{r}_{xe_F,t+1}\right] = 0 \tag{53}$$

$$E_t\left[(\hat{C}_{t+1} - \hat{C}^*_{t+1} - \hat{Q}_{t+1}/\rho)\hat{r}_{xb_H,t+1}\right] = 0 \tag{54}$$

In vector notation, these equations simplify to

$$E_t\left[(\hat{C}_{t+1} - \hat{C}^*_{t+1} - \hat{Q}_{t+1}/\rho)\hat{\mathbf{r}}_{x,t+1}\right] = \mathbf{0} \tag{55}$$

[22]Note that, here and in what follows, the order notation is suppressed. That is, the term $O(\epsilon^n)$, which is a residual that contains all terms of order n and higher, is omitted. When taking a second order approximation, e.g., it is the term $O(\epsilon^3)$ that is suppressed.

[23]For these variables we cannot compute the log deviation since their steady state value is zero.

where $\hat{\mathbf{r}}_{x,t+1}$ is a 3×1 vector of excess returns, that is, $\hat{\mathbf{r}}'_{x,t+1} = \begin{pmatrix} \hat{r}_{xe_H,t+1} & \hat{r}_{xe_F,t+1} & \hat{r}_{xb_H,t+1} \end{pmatrix}$. Here and in what follows, a variable set in bold face indicates that it is a vector or a matrix. Equation (55) implies that the covariance between the real exchange rate adjusted relative marginal utilities of consumption (henceforth referred to as relative marginal utilities, $cd_{t+1} \equiv \hat{C}_{t+1} - \hat{C}^*_{t+1} - \hat{Q}_{t+1}/\rho$) and the ex post excess return, $\hat{\mathbf{r}}_{x,t+1}$, is equal to zero. It captures differences between assets in their ability to hedge relative consumption risk and thus ties down an optimal portfolio allocation. Define $\boldsymbol{\alpha} \equiv \begin{pmatrix} \alpha_{e_H} & \alpha_{e_F} & \alpha_{b_H} \end{pmatrix}$ to be the home household's vector of steady state portfolio holdings. The choice of this vector affects the covariance between relative marginal utilities and excess returns and only if the optimal $\boldsymbol{\alpha}$ is chosen, this covariance is zero so that condition (55) holds.

Devereux and Sutherland (2011) highlight that in order to derive this optimal portfolio allocation three properties of the approximated model are relevant: First, and as evident in equation (55), to pin down the optimal portfolio allocation it is sufficient to derive the first-order component of relative marginal utilities and ex post excess returns, cd_{t+1} and $\hat{\mathbf{r}}_{x,t+1}$, respectively. From a first order approximation of the model, we can derive these terms and compute the covariance between the two. Second, the only component of the portfolio choice that affects cd_{t+1} and $\hat{\mathbf{r}}_{x,t+1}$, and hence, the covariance between the two, is the steady state portfolio allocation $\boldsymbol{\alpha}$. First- or higher-order components of $\boldsymbol{\alpha}$ have no effect. In the following, it is convenient to describe the solution procedure in terms of the solution for $\tilde{\boldsymbol{\alpha}}$, where $\tilde{\boldsymbol{\alpha}} \equiv \frac{1}{\beta Y}\boldsymbol{\alpha}$. $\tilde{\boldsymbol{\alpha}}$ solely enters the first order approximation of the model equations through the domestic household's budget constraint. The first order approximation of this equation (in real terms) reads (see Appendix D for a derivation)

$$N\hat{F}A_t = \frac{1}{\beta}N\hat{F}A_{t-1} + \hat{Y}_t - \hat{C}_t + \hat{P}_{H,t} - \hat{P}_t + \tilde{\boldsymbol{\alpha}}\hat{\mathbf{r}}_{x,t} \tag{56}$$

where $N\hat{F}A_t \equiv \frac{NFA_t}{Y}$ and Y is the steady state output level. We can see that the first order component of $\boldsymbol{\alpha}$ ($\hat{\boldsymbol{\alpha}}_t$) does not enter equation (56) and has therefore no effect on the first order approximation of the model. The reason is that in the steady state, excess returns are zero, $r_{xk} = 0 \quad \forall k \in K'$, and hence, the term in the first order approximation of the budget constraint that contains $\hat{\boldsymbol{\alpha}}_t$ drops out (see equation (132) in Appendix D).

Third, the portfolio excess return $\tilde{\boldsymbol{\alpha}}\hat{\mathbf{r}}_{x,t}$ is – up to a first order approximation of the model – a zero-mean i.i.d. random variable. We can see this by adding up equations (46) and (49), (47) and (50), and (48) and (51), which yields

$$E_t(\hat{r}_{xe_H,t+1}) = \frac{\sigma}{2}E_t[\hat{r}_{xe_H,t+1}(\hat{C}_{t+1} + \hat{C}^*_{t+1} + \frac{1}{\sigma}\hat{Q}_{t+1})] - \frac{1}{2}E_t(\hat{r}^2_{e_H,t+1} - \hat{r}^2_{b_F,t+1}) \tag{57}$$

$$E_t(\hat{r}_{xe_F,t+1}) = \frac{\sigma}{2}E_t[\hat{r}_{xe_F,t+1}(\hat{C}_{t+1} + \hat{C}^*_{t+1} + \frac{1}{\sigma}\hat{Q}_{t+1})] - \frac{1}{2}E_t(\hat{r}^2_{e_F,t+1} - \hat{r}^2_{b_F,t+1}) \tag{58}$$

$$E_t(\hat{r}_{xb_H,t+1}) = \frac{\sigma}{2}E_t[\hat{r}_{xb_H,t+1}(\hat{C}_{t+1} + \hat{C}^*_{t+1} + \frac{1}{\sigma}\hat{Q}_{t+1})] - \frac{1}{2}E_t(\hat{r}^2_{b_H,t+1} - \hat{r}^2_{b_F,t+1}) \tag{59}$$

Since only second-order terms enter the set of equilibrium expected excess returns, up to a first order approximation, $E_t(\hat{\mathbf{r}}_{x,t+1})$ is zero and there is no predictable element in $\hat{\mathbf{r}}_{x,t+1}$. Therefore, and since $\tilde{\boldsymbol{\alpha}}$ is time invariant, the portfolio excess return $\tilde{\boldsymbol{\alpha}}\hat{\mathbf{r}}_{x,t}$ is a function of the i.i.d. stochastic shocks in the model and must itself be an i.i.d. random variable.

Keeping these three properties in mind the solution process to find $\tilde{\boldsymbol{\alpha}}$ can be described with the following three steps: Step 1, the portfolio excess return $\tilde{\boldsymbol{\alpha}}\hat{\mathbf{r}}_{x,t}$ in equation (56) is replaced by ζ_t and temporarily treated as an exogenous i.i.d. random variable. Using standard linear rational expectations solution procedures, all parts of the model apart from the portfolio allocation are then solved (conditional on a given value of $\tilde{\boldsymbol{\alpha}}$[24]) to yield a state space solution.

Step 2, we extract the necessary information from the estimation in Step 1 to derive the optimal $\tilde{\boldsymbol{\alpha}}$. Suppose that the state space solution looks as follows

$$\mathbf{s}_{t+1} = \mathbf{F}_1\mathbf{x}_t + \mathbf{F}_2\mathbf{s}_t + \mathbf{F}_3\zeta_t \tag{60}$$

$$\mathbf{c}_t = \mathbf{P}_1\mathbf{x}_t + \mathbf{P}_2\mathbf{s}_t + \mathbf{P}_3\zeta_t \tag{61}$$

The vector $\mathbf{s}$ contains predetermined variables, $\mathbf{c}$ is the vector of forward-looking (jump) variables and $\mathbf{x}$ is the vector of exogenous variables, where $\mathbf{x}_t = N\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t$ and $\boldsymbol{\varepsilon}_t = \begin{pmatrix} u & u^* & m & m^* \end{pmatrix}'$. $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \mathbf{P}_1, \mathbf{P}_2$ and $\mathbf{P}_3$ are coefficient matrices. From this solution, we can extract the appropriate rows to find the behavior of cd_{t+1} and $\hat{\mathbf{r}}_{x,t+1}$.

$$\hat{\mathbf{r}}_{x,t+1} = \mathbf{R}_1\zeta_{t+1} + \mathbf{R}_2\boldsymbol{\varepsilon}_{t+1} \tag{62}$$

$$\hat{C}_{t+1} - \hat{C}^*_{t+1} - \hat{Q}_{t+1}/\rho = D_1\zeta_{t+1} + \mathbf{D}_2\boldsymbol{\varepsilon}_{t+1} + \mathbf{D}_3\begin{bmatrix}\mathbf{x}_t \\ \mathbf{s}_{t+1}\end{bmatrix} \tag{63}$$

where $\mathbf{R}_1, \mathbf{R}_2, D_1, \mathbf{D}_2$ and $\mathbf{D}_3$ are coefficient matrices contained in $\mathbf{P}_1, \mathbf{P}_2$ and $\mathbf{P}_3$.

Step 3, given equations (62) and (63), we can evaluate equation (55) and thus solve for the optimal portfolio allocation $\tilde{\boldsymbol{\alpha}}$. It is shown in Devereux and Sutherland (2011) that the solution for $\tilde{\boldsymbol{\alpha}}$ is

$$\tilde{\boldsymbol{\alpha}} = [\mathbf{R}_2\boldsymbol{\Sigma}\mathbf{D}_2'\mathbf{R}_1' - D_1\mathbf{R}_2\Sigma\mathbf{R}_2']^{-1}\mathbf{R}_2\boldsymbol{\Sigma}\mathbf{D}_2' \tag{64}$$

where $\boldsymbol{\Sigma}$ is the variance-covariance matrix of the vector of innovations $\boldsymbol{\varepsilon}$. The relevant coefficient matrices from equations (62) and (63) that affect $\tilde{\boldsymbol{\alpha}}$ are $\mathbf{R}_1$ and $\mathbf{R}_2$ (the responses of $\hat{\mathbf{r}}_{x,t+1}$ with respect to ζ_{t+1} and $\boldsymbol{\varepsilon}_{t+1}$, respectively), and D_1 and $\mathbf{D}_2$ (the responses of cd_{t+1} w.r.t. to ζ_{t+1} and $\boldsymbol{\varepsilon}_{t+1}$, respectively.

Finally, using standard methods, the non-portfolio parts of the model can be solved, this time conditional on the optimal portfolio allocation computed with equation (64). This will provide us with the policy and transition functions for all model variables under the optimal portfolio allocation.

3.3 Log-Linearization of the Model

As mentioned before, all parts of the model (apart from the portfolio allocation) are solved by first order approximations around the symmetric steady state. A summary of the first order

[24] The choice of this value does not affect the optimal allocation of holdings of equities and bonds (e_H, e_F, b_H and b_F).

Optimal consumption	$\rho(E_t\hat{C}_{t+1}-\hat{C}_t)=\rho(E_t\hat{C}^*_{t+1}-\hat{C}^*_t)+(2\mu-1)(E_t\hat{\tau}_{t+1}-\hat{\tau}_t)$
Home budget constraint	$N\hat{F}A_t=\frac{1}{\beta}N\hat{F}A_{t-1}+\hat{Y}_t-\hat{C}_t+\hat{P}_{H,t}-\hat{P}_t+\tilde{\boldsymbol{\alpha}}\hat{\mathbf{r}}_{x,t}$
Home output	$\hat{Y}_t=\mu\hat{C}_t+(1-\mu)\hat{C}^*_t-\theta\mu(\hat{P}_{H,t}-\hat{P}_t)-\theta(1-\mu)(\hat{P}_{H,t}-\hat{S}_t-\hat{P}^*_t)$
Foreign output	$\hat{Y}^*_t=\mu\hat{C}^*_t+(1-\mu)\hat{C}_t-\theta\mu(\hat{P}^*_{F,t}-\hat{P}^*_t)-\theta(1-\mu)(\hat{P}^*_{F,t}+\hat{S}_t-\hat{P}_t)$
Home PP inflation	$\pi_{H,t}=\frac{1}{\lambda}(\rho\hat{C}_t+\chi\hat{Y}_t+(1-\mu)\hat{\tau}_t-(1+\chi)\hat{A}_t)+\beta E_t\pi_{H,t+1}$
Foreign PP inflation	$\pi^*_{F,t}=\frac{1}{\lambda}(\rho\hat{C}^*_t+\chi\hat{Y}^*_t-(1-\mu)\hat{\tau}_t-(1+\chi)\hat{A}^*_t)+\beta E_t\pi^*_{F,t+1}$
Home CIA constraint	$\hat{M}_t-\hat{P}_t=\hat{C}_t$
Foreign CIA constraint	$\hat{M}^*_t-\hat{P}^*_t=\hat{C}^*_t$
Home consumer prices	$\hat{P}_t=\hat{P}_{H,t}+(1-\mu)\hat{\tau}_t$
Foreign consumer prices	$\hat{P}^*_t=\hat{P}^*_{F,t}-(1-\mu)\hat{\tau}_t$
Terms of trade	$\hat{\tau}_t=\hat{P}^*_{F,t}+\hat{S}_t-\hat{P}_{H,t}$

Table 1: Linear approximation of the non-portfolio parts of the model for a given $\tilde{\alpha}$. Note: A hat indicates the log deviation of a variable from its value in the non-stochastic steady state. $\tilde{\boldsymbol{\alpha}}\equiv\frac{1}{\beta Y}\boldsymbol{\alpha}$, where $\boldsymbol{\alpha}$ is a vector that contains the elements $\alpha_{e_H},\alpha_{e_F}$ and α_{b_H}. Y is the steady state output level. $\hat{\mathbf{r}}_{x,t}$ is a vector that contains the elements $\hat{r}_{xe_H,t}$, $\hat{r}_{xe_F,t}$ and $\hat{r}_{xb_H,t}$. $\pi_{H,t}$ and $\pi^*_{F,t}$ are the inflation rates of producer prices (PP), defined as $\pi_{H,t}=\hat{P}_{H,t}-\hat{P}_{H,t-1}$, and $\pi^*_{F,t}=\hat{P}^*_{F,t}-\hat{P}^*_{F,t-1}$ and $\lambda\equiv\frac{\kappa}{(1-\kappa)(1-\beta\kappa)}$. $\hat{A}_t$, $\hat{A}^*_t$, $\hat{M}_t$ and $\hat{M}^*_t$ are given by $\hat{A}_t=\psi_a\hat{A}_{t-1}+u_t$, $\hat{A}^*_t=\psi_a\hat{A}^*_{t-1}+u^*_t$, $\hat{M}_t=\psi_m\hat{M}_{t-1}+m_t$, and $\hat{M}^*_t=\psi_m\hat{M}^*_{t-1}-\epsilon\hat{S}_t+m^*_t$.

approximation of the non-portfolio parts of the model is provided in Table 1. The derivation of these equations and a first order approximation of all model equations can be found in Appendix D. Note that the non-portfolio parts of the model are very similar to the ones in Devereux and Sutherland (2008). Differences arise from the fact that Devereux and Sutherland (2008) set μ to 0.5 and use an interest rate rule instead of a money supply rule.[25]

To shed light on the transmission mechanism of productivity and monetary shocks in the model, we have to derive further analytical expressions in addition to the ones in Table 1. In a first step, we derive the relationship between the terms of trade and the real exchange rate by combining the equations that determine the real exchange rate, the terms of trade, and the home and foreign consumer prices (see Appendix D for a derivation). This yields

$$\hat{Q}_t=(2\mu-1)\hat{\tau}_t \tag{65}$$

stating that, due to the home bias in consumption ($\mu>0.5$), an improvement of the home terms of trade ($\tau\downarrow$) generates a real appreciation of the home currency ($Q\downarrow$), implying that the price of the foreign consumption basket in units of home consumption decreases.

In a next step, we derive the effect of changes in home relative to foreign output on the terms of trade. Combining the difference between home and foreign output with the equations that

[25] When $\mu=0.5$, the real exchange rate Q_t is always equal to one and $\hat{Q}_t$ is zero. Another difference is that I use a more general utility function by setting $\chi>0$ (as in Dellas and Tavlas (2013)), while Devereux and Sutherland (2008) set $\chi=0$, implying that the household's utility is linear in hours of labor.

determine the terms of trade and the home and foreign consumer prices yields

$$\begin{aligned}\hat{Y}_t - \hat{Y}_t^* &= \mu\hat{C}_t + (1-\mu)\hat{C}_t^* - \theta\mu(\hat{P}_{H,t} - \hat{P}_t) - \theta(1-\mu)(\hat{P}_{H,t} - \hat{S}_t - \hat{P}_t^*) \\ &\quad - (\mu\hat{C}_t^* + (1-\mu)\hat{C}_t - \theta\mu(\hat{P}_{F,t}^* - \hat{P}_t^*) - \theta(1-\mu)(\hat{P}_{F,t}^* + \hat{S}_t - \hat{P}_t)) \end{aligned} \tag{66}$$

$$\hat{Y}_t - \hat{Y}_t^* = (2\mu-1)(\hat{C}_t - \hat{C}_t^*) + 4\theta\mu(1-\mu)\hat{\tau}_t \tag{67}$$

$$\hat{Y}_t - \hat{Y}_t^* = (2\mu-1)(\hat{C}_t - \hat{C}_t^*) + \theta(1-(2\mu-1)^2)\hat{\tau}_t \tag{68}$$

This equation can be simplified further by using the risk-sharing condition that holds under complete markets. Under complete markets, agents can fully insure themselves against idiosyncratic (country-specific) risks and only face aggregate risks by trading financial assets. Home and foreign stochastic discount factors (i.e., home and foreign marginal utility growth) are then identical if both are measured in terms of the home consumption basket:[26]

$$SDF_{t+1} = SDF_{t+1}^* \frac{Q_t}{Q_{t+1}} \tag{69}$$

$$\Leftrightarrow \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho} = \beta\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\rho}\frac{Q_t}{Q_{t+1}} \tag{70}$$

where $SDF_{t+1}^* \equiv \beta\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\rho}$ is the foreign stochastic discount factor. Equation (70) is the Backus-Smith-Kollmann condition, which is the fundamental risk-sharing condition in the presence of real exchange rate fluctuations (see Backus and Smith (1993) and Kollmann (1995)). Consumption growth is not equal across countries (as would be the case when there were no real exchange rate fluctuations), but it is linked to changes in the real exchange rate. Define the impact period, i.e., the period when a shock occurs, to be period 1. Since, in steady state, countries are symmetric, i.e., $C = C^*$ and $Q = 1$, on impact we can rewrite the risk sharing condition as $C_1^{-\rho} = C_1^{*-\rho}\frac{1}{Q_1}$. Log-linearizing this condition and rearranging gives

$$\rho(\hat{C}_1 - \hat{C}_1^*) = \hat{Q}_1 \tag{71}$$

This implies that, if measured in terms of the home consumption basket, home and foreign households have the same marginal utilities on impact, i.e., relative marginal utilities cd_1 are zero. It also implies that consumption should be higher in the country where it is cheaper to consume. Substituting this equation into equation (68) and combining it with equation (65) yields the following relationship between changes in home relative to foreign output and the terms of trade:

$$\hat{Y}_1 - \hat{Y}_1^* = (2\mu-1)(\hat{C}_1 - \hat{C}_1^*) + \theta(1-(2\mu-1)^2)\hat{\tau}_1 \tag{72}$$

$$\Leftrightarrow \hat{Y}_1 - \hat{Y}_1^* = \left[(2\mu-1)^2\frac{1}{\rho} + \theta(1-(2\mu-1)^2)\right]\hat{\tau}_1 \tag{73}$$

$$\Leftrightarrow \hat{\tau}_1 = \frac{1}{\gamma}(\hat{Y}_1 - \hat{Y}_1^*) \tag{74}$$

[26]Under incomplete asset markets, they are only identical in expectations.

where $\gamma = [1-(2\mu-1)^2]\theta + \frac{(2\mu-1)^2}{\rho} > 0$.[27] With complete markets, the international transmission of productivity shocks is always positive: A relative increase in home output $((\hat{Y}_1 - \hat{Y}_1^*) \uparrow)$, which is observed when a home productivity shock occurs (independent of the degree of price flexibility), is associated with a worsening of the domestic terms of trade ($\tau \uparrow$), benefiting foreign consumers. In spite of this, the increase in consumption is higher at home than abroad since a home bias in consumption is assumed. This can be seen by rewriting equation (71) using equations (65) and (74), yielding

$$\hat{C}_1 - \hat{C}_1^* = \frac{(2\mu-1)}{\rho}\frac{1}{\gamma}(\hat{Y}_1 - \hat{Y}_1^*) \tag{75}$$

With a home bias in consumption, the elasticity term above is always positive. Therefore, even if the home terms of trade decline, this will never cause a decrease in home consumption – neither in absolute level, nor relative to foreign consumption (i.e., there is no immiserizing growth). In terms of magnitude, the effect of a change in relative output on the difference between home and foreign consumption will be larger the lower the elasticity of substitution among home and foreign goods θ (as θ increases the home and foreign good become closer substitutes and we approach a one good model). In the special case of identical home and foreign consumption baskets, i.e., $\mu = 0.5$, as in Devereux and Sutherland (2008), the effect on the difference between home and foreign consumption is zero.

In a next step, we derive the effect of changes in home relative to foreign output on the current account. A first order approximation of the current account yields

$$\hat{CA}_t = \hat{P}_{H,t} - \hat{P}_t + \hat{Y}_t - \hat{C}_t + N\hat{F}A_{t-1}\left(\frac{1-\beta}{\beta}\right) \tag{76}$$

where $\hat{CA}_t \equiv \frac{CA_t}{Y}$. Since net foreign assets are zero in the steady state, on impact the current account simplifies to

$$\hat{CA}_1 = \hat{P}_{H,1} - \hat{P}_1 + \hat{Y}_1 - \hat{C}_1 \tag{77}$$

which is equal to net exports. Since the home current account is equal to the negative of the foreign current account, we have that $\hat{CA}_1 = -\hat{CA}_1^*$, or, adding $\hat{CA}_1$ on each side of this

[27] See Corsetti et al. (2008), Coeurdacier and Rey (2013), Coeurdacier (2009) and Coeurdacier et al. (2009) for similar expressions.

equation, $2\hat{CA}_1 = \hat{CA}_1 - \hat{CA}_1^*$.[28] This allows to rewrite the current account as follows

$$\hat{CA}_1 = \frac{1}{2}[\hat{P}_{H,1} - \hat{P}_1 + \hat{Y}_1 - \hat{C}_1 - \hat{P}_{F,1} + \hat{P}_1^* - \hat{Y}_1^* + \hat{C}_1^*] \tag{78}$$

$$\Leftrightarrow \hat{CA}_1 = \frac{1}{2}[(\hat{Y}_1 - \hat{Y}_1^*) - (\hat{C}_1 - \hat{C}_1^*) + \hat{Q}_1 - \hat{\tau}] \tag{79}$$

$$\Leftrightarrow \hat{CA}_1 = \frac{1}{\gamma}(1-\mu)\left((\theta - 1) + (2\mu - 1)\left(\theta - \frac{1}{\rho}\right)\right)(\hat{Y}_1 - \hat{Y}_1^*) \tag{80}$$

stating that the current account (and net exports) is procyclical unless the elasticity of substitution between home and foreign goods θ is very low.

Next, we rewrite the equation for the nominal exchange rate using the money demand equations, equations (65), (74) and (75), to derive the relationship between relative output and relative monetary conditions on the one hand and the nominal exchange rate on the other hand:

$$\hat{S}_1 = \hat{Q}_1 - \hat{P}_1^* + \hat{P}_1 \tag{81}$$

$$\Leftrightarrow \hat{S}_1 = (2\mu - 1)\hat{\tau}_1 + (\hat{M}_1 - \hat{M}_1^*) - (\hat{C}_1 - \hat{C}_1^*) \tag{82}$$

$$\Leftrightarrow \hat{S}_1 = (2\mu - 1)\frac{1}{\gamma}\left(1 - \frac{1}{\rho}\right)(\hat{Y}_1 - \hat{Y}_1^*) + (\hat{M}_1 - \hat{M}_1^*) \tag{83}$$

This implies that, under a flexible exchange rate regime, a relative increase in home output (given that $\rho > 1$) or a relative increase in home money supply leads to a depreciation of the home currency ($S \uparrow$). Under a fixed exchange rate regime, in contrast, the foreign monetary authority insulates the nominal exchange rate from the effects of shocks by adjusting foreign money supply M^*.

So far, we have put little emphasis on the role of nominal rigidities. All equations above hold in both cases, when prices are flexible as well as when they are sticky. The magnitude of the response of home relative to foreign output $(\hat{Y}_t - \hat{Y}_t^*)$ to shocks, and hence the response of the other variables, however, will differ depending on the degree of price flexibility. Focusing on productivity shocks first, a positive home productivity shock, e.g., increases the supply of the home good and, in order to equilibrate demand and supply, the price of this good has to decrease. When prices are sticky not all firms can immediately adjust their price and therefore, on impact, aggregate home producer prices fall less compared to the decrease under flexible prices and output becomes demand-determined for small enough shocks. Thus, aggregate home output increases by less and the rise in home relative to foreign output $\hat{Y}_t - \hat{Y}_t^*$ is smaller than under flexible prices. As a consequence, the response of the other variables discussed above (terms of trade, relative consumption, current account) is smaller, too.

For the effect of monetary shocks, the degree of price flexibility matters too. Under flexible prices, a positive home monetary shock leads to an increase in nominal prices in the home country and the nominal exchange rate adjusts such that the price of home relative to foreign goods is

[28] Note that the real exchange rate does not enter this equation although it does appear in the level equation. In levels we $CA_1 = -CA_1^* Q_1$, taking a first order approximation yields $\hat{CA}_1 = -Q\hat{CA}_1^* - \frac{CA^* Q}{Y}\hat{Q}_1$, where Q, CA^* and Y are steady state values. Since in the steady state $Q = 1$ and $CA^* = 0$, this equation simplifies to $\hat{CA}_1 = -\hat{CA}_1^*$.

restored. Hence, the home consumer price level and the nominal exchange rate adjust one to one with the change in money supply and money has no effect on the real economy, i.e., $\hat{Y}_1 - \hat{Y}_1^*$ is zero. When prices are sticky, in contrast, only a fraction of firms can raise their price and an increase in money supply does not lead to a one to one increase in aggregate home producer and consumer prices, resulting in a rise in real money balances. Clearing of the home money market requires an increase in home consumption[29], raising short-run home output. Hence, money is non-neutral and has real effects. Similar to a positive home productivity shock, under sticky prices, a positive home monetary shock raises home relative to foreign output $\hat{Y}_1 - \hat{Y}_1^*$.

In a final step, we derive the equations that determine prices of real and nominal assets, the variables on whose shock responses the main analysis of this paper will focus. According to the central asset pricing formula, real asset prices for equity and bonds are given by (see Appendix F for the derivation)

$$\hat{Z}_{e,t} = (1-\beta)E_t\hat{\Pi}_{t+1} + \beta E_t\hat{Z}_{e,t+1} - \rho(E_t\hat{C}_{t+1} - \hat{C}_t) \tag{84}$$

$$\hat{Z}^*_{e,t} = (1-\beta)E_t\hat{\Pi}^*_{t+1} + \beta E_t\hat{Z}^*_{e,t+1} - \rho(E_t\hat{C}^*_{t+1} - \hat{C}^*_t) \tag{85}$$

$$\hat{Z}_{b,t} = -E_t\hat{P}_{t+1} - \rho(E_t\hat{C}_{t+1} - \hat{C}_t) \tag{86}$$

$$\hat{Z}^*_{b,t} = -E_t\hat{P}^*_{t+1} - \rho(E_t\hat{C}^*_{t+1} - \hat{C}^*_t) \tag{87}$$

where the linearized versions of the home and foreign stochastic discount factors can be defined as $S\hat{D}F_{t+1} \equiv -\rho(\hat{C}_{t+1} - \hat{C}_t)$ and $S\hat{D}F^*_{t+1} \equiv -\rho(\hat{C}^*_{t+1} - \hat{C}^*_t)$. Generally, real asset prices are forward looking and depend on future payoffs and the future stochastic discount factor. Although the asset pricing equations of the respective country only contain the domestic stochastic discount factor, the discount factor of the other country also plays a role. The reason for this is that the two discount factors are related via the (log-linearized) fundamental risk sharing condition under complete markets, $S\hat{D}F_{t+1} = S\hat{D}F^*_{t+1} + \hat{Q}_t - \hat{Q}_{t+1}$.

Real prices of home and foreign equity can be iterated forward in time to find

$$\hat{Z}_{e,t} = (1-\beta)E_t\sum_{i=0}^{\infty}\beta^i\hat{\Pi}_{t+1+i} + E_t\sum_{i=0}^{\infty}\beta^i S\hat{D}F_{t+1+i} \tag{88}$$

$$\hat{Z}^*_{e,t} = (1-\beta)E_t\sum_{i=0}^{\infty}\beta^i\hat{\Pi}^*_{t+1+i} + E_t\sum_{i=0}^{\infty}\beta^i S\hat{D}F^*_{t+1+i} \tag{89}$$

These equations show that equity prices are determined by the expected present value of future net cash flows (dividends). Changes in real equity prices depend on changes in future dividends and stochastic discount factors. Note that, if $\sum_{i=0}^{\infty}\beta^i S\hat{D}F_{t+1+i}$, e.g., is positive, it indicates that due to the decline in future home consumption, the home household has a higher appetite for wealth in the future relative to the present period and that the rate at which he is willing to substitute future consumption for present consumption is higher. This has a positive effect on the present home equity price.

[29] The cash-in-advance constraint is always binding since the nominal interest rate (R_t) is positive and in this case the opportunity costs of holding money unspent are too high.

Given that *nominal* asset prices play a major role in international financial markets, the main analysis of this paper will focus on asset prices in nominal instead of real terms. The nominal price of equity equals its real price plus respective contemporaneous inflation.[30] On impact we have

$$\hat{Q}_{e,1} = (1-\beta)E_1 \sum_{i=0}^{\infty} \beta^i \hat{\Pi}_{2+i} + E_1 \sum_{i=0}^{\infty} \beta^i S\hat{D}F_{2+i} + \hat{P}_1 \tag{90}$$

$$\hat{Q}^*_{e,1} = (1-\beta)E_1 \sum_{i=0}^{\infty} \beta^i \hat{\Pi}^*_{2+i} + E_1 \sum_{i=0}^{\infty} \beta^i S\hat{D}F^*_{2+i} + \hat{P}^*_1 \tag{91}$$

For nominal home and foreign currency bond prices, we have on impact:[31]

$$\hat{Q}_{b,1} = -(E_1\hat{P}_2 - \hat{P}_1) - \rho(E_t\hat{C}_2 - \hat{C}_1) \tag{92}$$

$$\hat{Q}^*_{b,1} = -(E_1\hat{P}^*_2 - \hat{P}^*_1) - \rho(E_t\hat{C}^*_2 - \hat{C}^*_1) \tag{93}$$

As we can see, nominal bond prices are determined by future inflation and the future stochastic discount factor, where in this case when referring to "future" we mean the next period. Next period's home and foreign consumer price inflation are defined to be $\pi_2 = \hat{P}_2 - \hat{P}_1$ and $\pi^*_2 = \hat{P}^*_2 - \hat{P}^*_1$, respectively.[32] Future inflation has a negative effect on the nominal bond price and a positive stochastic discount factor has a positive effect. If $S\hat{D}F_2$ is, e.g., positive, this implies that home consumption in period two falls, i.e., $C_2 < C_1$. Then, the home household has a higher appetite for wealth in period two and higher incentives to invest in home bonds, which positively affects the home bond price on impact.

4 Results

In this section, the optimal portfolio allocation is explained and derived. Furthermore, the responses of key macroeconomic variables and asset prices to productivity and monetary shocks when agents hold this optimal portfolio are presented. In subsections 4.1 and 4.2, this is done first for the case of a flexible exchange rate regime, before, in subsections 4.3 to 4.5, the same exercise is conducted for the case of a fixed exchange rate regime.

The numerical results are computed using Dynare. The parametrization I apply is shown in Table 2. It is similar to the one in Dellas and Tavlas (2013), but differs from the latter in

[30] Alternatively, one could use the equity pricing formula in nominal terms. The price of home equity, e.g., is then given by the covariance of the nominal stochastic discount factor, $\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\frac{P_t}{P_{t+1}}$, and the nominal payoff, $\Pi_{t+1}P_{t+1} + Q_{e,t+1}$. However, for real shocks it is more interesting to analyze the effect on real dividends and real stochastic discount factors.

[31] For home and foreign currency bonds, it is less interesting to analyze the effect of a shock on the real price and therefore we directly focus on nominal prices.

[32] Here the same definition as Devereux and Sutherland (2008) use for producer price inflation is applied. Note that this definition slightly deviates from the standard way of defining inflation (an exception is the impact period, where $\pi_1 = \hat{P}_1$, then the two are identical). π_2 can be rewritten as follows: $\pi_2 = \hat{P}_2 - \hat{P}_1 = \frac{P_2 - P}{P} - \frac{P_1 - P}{P} = \frac{P_2 - P_1}{P}$, while the usual definition is $\frac{P_2 - P_1}{P_1}$. Thus, the signs will always be the same, but the values will slightly differ across the two definitions.

the following ways. First, since in my model prices are assumed to be sticky à la Calvo, the set of parameters contains an additional parameter of price stickiness κ. As in Gali (2008), I set $\kappa = \frac{2}{3}$, implying that only a fraction $1 - \kappa = \frac{1}{3}$ of firms reset their price in every period. Given a quarterly frequency of the model, this translates into an average price duration of three quarters, a value consistent with empirical evidence.

Parameter	Description	Value
β	Discount factor	0.96
ρ	Consumption curvature	2.00
χ	Labor supply curvature	1.00
v	Velocity of money	0.50
θ	Elasticity of substitution between home and foreign goods	1.10
ϕ	Elasticity of substitution between individual goods	6.00
μ	Share of local goods in the consumption basket	0.75
$\mathcal{M}$	Mark up	1.20
κ	Index of price stickiness	0.66
ψ_a, ψ_m	Persistence of productivity and monetary shocks	0.90
σ_a, σ_m	Standard deviation of productivity and monetary shocks	0.01

Table 2: Parameters

Second, I choose a lower elasticity of substitution between home and foreign goods ($\theta = 1.1$ versus $\theta = 4$), implying that home and foreign goods are less close substitutes. As argued in Devereux and Sutherland (2008), for any positive κ, a value of θ close enough to unity results in a equity home bias.[33] Such a bias is consistent with empirical evidence.[34]

Finally, exchange rate flexibility is measured by parameter ϵ, which equals zero in the case of completely flexible exchange rates.

4.1 Flexible Exchange Rate Regime: Steady State Portfolio Allocation

As outlined in the previous section, the steady state portfolio represents a solution to the following orthogonality condition:

$$E_t[cd_{t+1}\hat{\mathbf{r}}_{x,t+1}] = \mathbf{0} \tag{94}$$

The optimal portfolio allocation is one where fluctuations in relative marginal utilities are zero ($cd_{t+1} = 0$) for every possible realization of shocks. That is, the Backus-Smith-Kollmann condi-

[33]There are alternative ways to generate a equity home bias. One could, e.g., introduce transaction costs as in Tille and van Wincoop (2010).

[34]Despite the increase in cross-border financial trade over the last 30 years, international portfolios remain heavily tilted toward domestic assets. It is a general phenomenon that foreign equities comprise a small proportion of investors' portfolios, reflecting a equity home bias. The share of US stocks in US investors' equity portfolios, e.g., was 77.2% in 2008 (see Coeurdacier and Rey (2013) for a recent survey).

tion holds (see equation (71)) and there is full risk insurance across countries. If agents hold a portfolio different from this optimal portfolio, the covariance between relative marginal utilities and excess returns is not zero and the Backus-Smith-Kollmann condition does not hold.

In general, due to the fact that the two countries are symmetric, it will always be the case that the home agent holds the same amount of home equity as the foreign agent holds foreign equity. Equivalently, the home agent will always hold the same amount of home bond holdings as the foreign household holds foreign bond holdings. The optimal portfolio allocation will therefore always be one where $\tilde{\alpha}_{e_H} = \tilde{\alpha}^*_{e_F}$, $\tilde{\alpha}_{e_F} = \tilde{\alpha}^*_{e_H}$ and $\tilde{\alpha}_{b_H} = \tilde{\alpha}^*_{b_F}$. Together with the market clearing conditions (equation (42)), this implies that (a) holdings of home equity equal the negative of holdings of foreign equity, i.e., $\tilde{\alpha}_{e_H} = -\tilde{\alpha}_{e_F}$, and (b) holdings of home bonds equal the negative of holdings of foreign bonds, i.e., $\tilde{\alpha}_{b_H} = -\tilde{\alpha}_{b_F}$.

In order to find the motive for a particular portfolio allocation, we need to investigate the degree to which each asset is useful in hedging against relative consumption risk resulting from the four shocks in the model. This can be done by looking at the (non-zero) covariance between relative marginal utilities and ex post excess returns when there is no trade in financial assets. Such a lack of trade implies that home and foreign households hold no bonds and do not transfer any of their claims to their firms' profits to the other country, that is, they hold 100% of the claims on the firms in their own country. Define this portfolio allocation to be the zero portfolio $\tilde{\boldsymbol{\alpha}}^0 = \mathbf{0}$.

Under the assumption that agents hold this zero portfolio, Table 3 shows the response of relative marginal utilities and asset returns to productivity and monetary shocks on impact.

Focusing on productivity shocks first, the results suggest that a positive *home* productivity shock, captured by a positive value of u, leads to an increase in relative marginal utilities. Therefore, to hedge against this risk, the home household should hold an asset whose realized return (or payoff) has a negative correlation with home productivity. Since a productivity shock yields an increase in the realized return on all four assets[35], the correlation between home productivity and all asset returns is positive. Thus, holding a negative position in one of the four assets represents an opportunity for home households to hedge against the relative consumption risk associated with a home productivity shock.

A positive *foreign* productivity shock, captured by a positive value of u^*, has symmetric effects. It leads to a decrease in relative marginal utilities and an increase in all asset returns. Against such a shock, households can hedge by holding a positive position in one of the four assets.

[35]Realized asset returns depend on contemporaneous payoffs (see equations (140)-(143) in Appendix D). The return on home equity increases, since the shock directly raises the dividend payment of the home firm. Besides the dividend, the asset price, i.e., the second component of the payoff, increases on impact, too. The return on foreign equity rises due to the higher real asset price, which more than compensates for the lower dividend payments. In addition, the increase in the real exchange rate contributes to a higher return measured in units of home consumption. The realized returns on the holdings of home and foreign bonds rise due to the fall in the home and foreign consumer price levels which result in higher home and foreign bond payoffs, respectively.

	cd	$\hat{r}_{e_H}$	$\hat{r}_{e_F}$	$\hat{r}_{b_H}$	$\hat{r}_{b_F}$	$\hat{r}_{e_H} - \hat{r}_{e_F}$	$\hat{r}_{b_H} - \hat{r}_{b_F}$
u	0.070	0.686	0.284	0.234	0.255	0.403	-0.020
u^*	-0.070	0.104	0.507	0.075	0.055	-0.403	0.020
m	0.013	0.202	0.684	-0.634	0.535	-0.482	-1.170
m^*	-0.013	0.318	-0.164	0.170	-1.000	0.482	1.170

Table 3: Responses of relative marginal utilities and asset returns to shocks under a flexible exchange rate regime, given that agents hold the zero portfolio $\tilde{\boldsymbol{\alpha}}^0$. For returns the values represent the percentage deviation from the steady state in response to a shock of one standard deviation. For *cd* and excess returns within asset classes, $\hat{r}_{e_H} - \hat{r}_{e_F}$ and $\hat{r}_{b_H} - \hat{r}_{b_F}$, the values represent differences in percentage points.

To hedge against *home and foreign* productivity shocks at the same time, the home household needs to trade at least two financial assets. Table 3 indicates that there exist two portfolio allocations in two assets with which the home household can insure against both, home and foreign productivity shocks. These two portfolios are the only ones that provide portfolio returns of same size and opposite sign across shocks. The first portfolio consists of a positive position in foreign equity $\tilde{\alpha}_{e_F} > 0$, matched by the same negative position in home equity $\tilde{\alpha}_{e_H} = -\tilde{\alpha}_{e_F}$ and zero bond holdings $\tilde{\alpha}_{b_H} = \tilde{\alpha}_{b_F} = 0$. The second portfolio consists of a positive position in home bonds $\tilde{\alpha}_{b_H} > 0$, an equivalent negative position in foreign bonds $\tilde{\alpha}_{b_F} = -\tilde{\alpha}_{b_H}$ and zero equity holdings $\tilde{\alpha}_{e_H} = \tilde{\alpha}_{e_F} = 0$. Both portfolio allocations provide the home household with a negative portfolio return when a home productivity shock realizes and a positive return of same size (due to symmetry) when an equivalent foreign productivity shock occurs (see excess returns within asset classes $\hat{r}_{e_H} - \hat{r}_{e_F}$ and $\hat{r}_{b_H} - \hat{r}_{b_F}$). By optimally choosing the size of each portfolio allocation, the home household can avoid the increase in relative marginal utilities in response to a positive home and the equivalent decrease in response to a positive foreign productivity shock.[36] Hence, both portfolios provide full insurance against productivity shocks. Combinations of these two portfolios constitute further optimal portfolio allocations.

In a next step, we assess which assets serve as possible hedges against monetary shocks. The results in Table 3 suggest that a positive *home* monetary shock, captured by a positive value of m, results (due to the sluggish price adjustment) in an increase in relative marginal utilities. To hedge against this risk, the home household should hold an asset whose return has a negative correlation with home money supply. Changes in money supply do, due to non-neutrality of money, affect real returns on all four assets.[37] Except of the return on the home bond, all asset

[36]Note that the size of this portfolio cannot be determined by looking at the values in Table 3, because the response of relative marginal utilities and asset returns to disturbances change when the home household chooses a portfolio different from the zero portfolio.

[37]Under flexible prices, a change in money supply does only lead to a change in the real return on the nominal bond in the respective country. Note that despite of this effect, money is completely neutral and has no effect on the real economy. The reason is that in equilibrium the bond markets do not exist, a result that also Devereux and Sutherland (2008) find. Agents hold no nominal bonds since they would unnecessarily expose themselves to monetary shocks and the resulting relative consumption risk, while they can insure themselves against productivity shocks by holding a portfolio consisting of equities only (money does not affect equity returns).

returns increase on impact. The former falls due to the increase in the home consumer price level P (in response to a home productivity shock P decreases), which negatively affects the bond's payoff in real terms. Therefore, the home household can insure against relative consumption risk coming from a home monetary shock by holding a positive position in home bonds or a negative position in one of the other three assets.

A positive *foreign* monetary shock leads to a decrease in relative marginal utilities. The returns on both home assets (equities and bonds) increase while the returns on both foreign assets (equities and bonds) decrease. The reason for the decline in the foreign equity return is the decrease in the real exchange rate. Therefore, holding a positive position in home assets or a negative position in foreign assets represents a possibility for the home household to hedge against the relative consumption risk coming from a foreign monetary shock.

To hedge against *home and foreign* monetary shocks at the same time, the home household can hold either an equity portfolio consisting of a positive position in home equity matched by the same negative position in foreign equity, or a bond portfolio consisting of a positive position in home bonds and an equivalent negative position in foreign bonds, or a combination of the two.

Finally, we assess the portfolio allocation that is optimal to hedge against *all four shocks* at the same time. For full risk insurance the home household needs to trade all four assets. As evident from above, the signs of home and foreign equity holdings in the equity portfolio that is optimal to hedge against monetary shocks are the opposite to those in the equity portfolio that is optimal to hedge against productivity shocks. Therefore, if the home household chooses an equity allocation to eliminate relative consumption risk associated with productivity (monetary) shocks, he faces an even larger change in relative marginal utilities when a monetary (productivity) shock occurs.

In contrast to the equity allocation, a bond allocation serves (to some extent) as a hedge against all four shocks: holding a positive position in home bonds and an equivalent negative position in foreign bonds provides insurance against both the risk coming from productivity and from monetary shocks. This is because both types of shocks have a similar effect on the nominal exchange rate S against which the mentioned bond allocation provides a hedge. The return of this bond portfolio is proportional to the excess return on the home bond, which, in turn, is equal to the unanticipated change in the nominal exchange rate $\hat{r}_{xb_H,t+1} = -(\hat{S}_{t+1} - E_t\hat{S}_{t+1})$ (for a derivation see Appendix F). Since the expectation about future exchange rate changes from the perspective of the steady state is zero, this excess return simplifies to $\hat{r}_{xb_H} = -\hat{S}$ on impact.

The optimal size of the mentioned bond allocation, however, will not be the same for hedging against productivity shocks as for hedging against monetary shocks. While the change in relative marginal utilities is larger than the change in the excess home bond return in response to productivity shocks, it is smaller than the change in the excess home bond return in response to monetary shocks. Therefore, the optimal bond portfolio to insure against productivity shocks is larger in size relative to the bond portfolio that is optimal to insure against monetary shocks. However, given a certain size of the bond portfolio, the household can insure against the remaining relative consumption risk by trading equity holdings. Holding a positive position in foreign equity (with an equivalent negative position in home equity) in addition to the bond portfolio increases the insurance against productivity shocks and reduces the insurance against monetary shocks, and thereby, it reduces the difference of the bond portfolio's optimal size across types of shocks. By optimally choosing the amount of bonds and equities, the home household can avoid fluctuations in relative marginal utilities under every possible realization of shocks, i.e., there is full risk insurance across countries.

To conclude, the optimal diversification strategy for home households to avoid fluctuations in relative marginal utilities consists of a portfolio with a positive position in foreign equity matched by the negative of this in home equity, combined with a positive position in home bonds and an equivalent negative position in foreign bonds. The numerical result can be derived using equation (64): the optimal portfolio allocation under a flexible exchange rate regime is $\tilde{\boldsymbol{\alpha}}^{\text{flex}} = \begin{pmatrix} -1.58 & 1.58 & 0.76 \end{pmatrix}$. Equity holdings provide insurance against productivity shocks, while bond holdings provide insurance against both productivity and monetary shocks. As mentioned before, these values reflect the home household's real external asset holdings. Real asset holdings of the home household, e_H, e_F, b_H and b_F, can be derived from $\tilde{\boldsymbol{\alpha}}^{\text{flex}}$ using, e.g., the definition of α_{e_H} (equation (6)), the fact that $\tilde{\alpha}_{e_H} = \frac{1}{\beta Y}\alpha_{e_H}$ and the steady state values of the real equity price and home output. This yields $e_H = 0.62$, which implies that the home household holds 62% of home equity under a flexible exchange rate regime. Therefore, there is a bias towards home equity. In the same way we find $e_F = 0.38$, $b_H = 0.38$ and $b_F = -0.38$.

4.2 Flexible Exchange Rate Regime: Shock Responses

In this subsection, we asses the impact of productivity and monetary shocks on key economic variables and asset prices, when the home household holds the optimal portfolio allocation $\tilde{\boldsymbol{\alpha}}^{\text{flex}}$ and the exchange rate is flexible. The results are provided in Table 4.

Focusing on productivity shocks first, the results suggest that a positive *home* productivity shock raises domestic output and consumption, increases foreign consumption and reduces foreign output. The higher supply of the domestic good leads to a fall in the relative price of this good and causes a deterioration of the domestic terms of trade (see equation (74)). Even though foreign consumers benefit from this change in terms of trade, consumption grows more at home than abroad due to the home bias in consumption. Given that the elasticity of substitution between home and foreign goods, θ, is sufficiently large, net exports rise and the home country runs a positive current account (see equation (80)).[38][39] Hence, the increase in home consumption is smaller than the rise in output, with the difference accounted for by exports to the foreign

[38]The current account is positive for $\theta > \frac{5}{6}$, given that for the remaining parameters in equation (80) the values in Table 2 are used.

[39]While the current account is positive, the change in net foreign assets $\varDelta NFA$ is negative. This discrepancy reflects valuation effects resulting from portfolio diversification. We have $\varDelta \hat{NFA}_t = \hat{CA}_t + \tilde{\boldsymbol{\alpha}}\hat{\mathbf{r}}_{x,t}$ (see equation (139) in Appendix D). Under a zero portfolio, valuation effects are zero ($\tilde{\boldsymbol{\alpha}}\hat{\mathbf{r}}_{x,t} = 0$) and the change in net foreign assets is identical to the current account. However, since the home country chooses the optimal portfolio $\tilde{\boldsymbol{\alpha}}^{\text{flex}}$, it suffers a capital loss on its portfolio when a home productivity shock realizes, i.e., $\tilde{\boldsymbol{\alpha}}\hat{\mathbf{r}}_{x,t} < 0$. This capital loss is larger than the positive movement in the trade account and therefore leads to a decrease in the home country's net foreign assets.

	$\hat{Y}$	$\hat{Y}^*$	$\hat{C}$	$\hat{C}^*$	$\hat{CA}$	$\hat{\tau}$	$\hat{S}$	$\hat{Q}$
u	0.368	-0.059	0.211	0.099	0.045	0.450	0.112	0.225
u^*	-0.059	0.368	0.099	0.211	-0.045	-0.450	-0.112	-0.225
m	0.623	-0.088	0.361	0.174	0.075	0.749	1.187	0.374
m^*	-0.088	0.623	0.174	0.361	-0.075	-0.749	-1.187	-0.374

	$\hat{M}$	$\hat{M}^*$	$\hat{P}$	$\hat{P}^*$	$\hat{P}_H$	$\hat{P}_F^*$	$\hat{\Pi}$	$\hat{\Pi}^*$
u	0.000	0.000	-0.211	-0.099	-0.324	0.014	5.741	-0.076
u^*	0.000	0.000	-0.099	-0.211	0.014	-0.324	-0.076	5.741
m	1.000	0.000	0.639	-0.174	0.452	0.013	-7.230	-0.268
m^*	0.000	1.000	-0.174	0.639	0.013	0.452	-0.268	-7.230

	$\hat{Z}_e$	$\hat{Z}_e^*$	$\hat{Q}_e$	$\hat{Q}_e^*$	$\hat{Q}_b$	$\hat{Q}_b^*$
u	0.478	0.109	0.267	0.011	-0.084	-0.037
u^*	0.109	0.478	0.011	0.267	-0.037	-0.084
m	0.512	0.342	1.150	0.168	0.289	0.094
m^*	0.342	0.512	0.168	1.150	0.094	0.289

Table 4: Responses to shocks under a flexible exchange rate regime, given that the home agent holds $\tilde{\boldsymbol{\alpha}}^{\text{flex}}$. The values represent the percentage deviation from the steady state in response to a shock of one standard deviation ($\sigma_a = \sigma_m = 0.01$). If there is, e.g., a positive home productivity shock of one standard deviation, then Y_t rises by 0.37% on impact. An exception is the current account, $\hat{CA}$, which is defined to be the absolute change from the steady state as a fraction of the steady state output level.

country. Foreign households earn a positive return on their portfolio, which is a second reason for the rise in their wealth. Overall, the wealth effect, which induces the foreign household to work less, is stronger than the substitution effect away from leisure to consumption, leading to a slight fall in foreign output and a marginal rise in the price of the foreign good.[40] The increase in home productivity causes a rise in the real exchange rate (real depreciation) and hence, also an increase in the nominal exchange rate S, that is, the domestic currency depreciates (see equation (83)).

To understand the impact of a positive productivity shock on asset prices, we have to keep in mind that asset prices are forward looking (see equations (90)-(93)), so that their response on impact does not only depend on the economy's contemporaneous but also its future response to a shock. Therefore, we first discuss the shock response of a variable whose future values affect all asset prices: consumption (home and foreign). Future consumption affects asset prices on impact via its effect on the future stochastic discount factor. To understand the impulse response of consumption, consider first the response of home output, which – together with the response of consumption and other selected variables – is provided in Figure A.1 in Appendix G. Price stickiness in combination with a persistent productivity shock ($\rho_a = 0.9$) result in a hump-

[40] The shift in world demand away from foreign to home goods caused by the fall in the relative price of home goods is not strong enough to induce a fall in P_F^*. The reason for this is the low elasticity of substitution between home and foreign goods.

shaped impulse response function for output. As mentioned before, when home productivity increases, the supply of the home good increases and the price of this good has to decrease. However, due to price stickiness, not all firms can adjust their price immediately. Aggregate home producer prices decrease less on impact compared to the fall under flexible prices, and aggregate production of the home good increases by less. Because of the high persistence of the shock, it is optimal for firms to lower their price (if they can) not only on impact, but also in subsequent periods, so that demand increases further and hence, home output increases further after impact, too. In this way, the staggered price adjustment translates in a staggered adjustment of production.[41] After a few periods, the decrease in productivity (the shock is temporary) is strong enough so that for firms that can adjust their price it is optimal to increase it. From this period on, the economy returns back to the steady state. The hump-shaped impulse response of output leads to a similar pattern in the response of several other model variables, including home and foreign consumption. Therefore, also consumption is not only rising on impact but in subsequent periods, too, before reverting to the steady state. As a consequence, the home and foreign stochastic discount factors are negative on impact, continue being negative in the periods after (in which consumption is still rising), and become positive as soon as the economy is reverting to the steady state.

Consider now the response of bond and equity prices to a home productivity shock. Focusing on the bond market first, both home and foreign nominal bond prices, Q_b and Q_b^*, decrease when home productivity rises. On the one hand, bond prices are negatively affected by the negative future stochastic discount factor, that is, households have fewer incentives to invest in bonds on impact because consumption in the next period is expected to be higher. On the other hand, the anticipated decline in the future price level ($\pi_2, \pi_2^* < 0$ due to the hump-shaped impulse response of P and P^*, respectively) raises the bond's payoff and thereby has a positive effect on its price. On aggregate, the effect of the negative future stochastic discount factor is stronger, so that the net change in the bond's price is negative. The fall of the home relative to the foreign bond price is sharper since the home economy and hence, home consumption, is more heavily affected by the shock.

Focusing on the equity market, both home and foreign nominal equity prices, Q_e and Q_e^*, rise in response to a home productivity shock. As outlined in Section 3.3, changes in the real equity prices on impact (Z_e and Z_e^*) depend on the expected sum of future stochastic discount factors and future changes in dividends. Nominal prices are in addition affected by inflation on impact. The sign of the sum of future stochastic discount factors (home and foreign) is positive and opposite to the sign of the stochastic discount factor on impact. The latter follows from the fact that shocks are temporary. Since consumption rises on impact and will return to the steady state level, the sum of all future changes in consumption is negative, resulting in a positive sum

[41] When prices are flexible, in contrast, firms can fully adjust their price when a shock occurs and therefore economic variables start their return to the steady state after impact, so that the impulse response function for output and many other variables are not hump-shaped.

of future stochastic discount factors.[42] Because of this decline in future consumption, agents have a higher appetite for wealth in the future relative to the impact period, which has a positive effect on equity prices on impact. Expected future profits of home firms are (on average) higher than before the shock and have a positive effect on the home equity price, too. Expected future profits of foreign firms are only marginally higher (see Π and Π^* in Figure A.1 in Appendix G). Compared to real equity prices, nominal equity prices increase less. This is due to the decrease in the consumer price level on impact. Were price levels unaffected by the shock, nominal equity prices would increase one to one with the increase in real prices. Compared to the foreign nominal equity price, the home equity price increases stronger because the home economy is more heavily affected by the home productivity shock. The increase of the home equity price is mainly driven by the increase in future home profits, while the rise of the foreign equity price is mainly caused by positive future foreign stochastic discount factors.

The effects of a *foreign* productivity shock are symmetric to the ones of a home productivity shock just described.

In a next step, we focus on the impact effects of a positive *home monetary* shock. Since prices are sticky, not all firms can raise their price and an increase in money supply does not yield a one to one increase in aggregate home producer and consumer prices (as under flexible prices), resulting in a rise in real money balances. Clearing of the home money market requires an increase in home consumption. Since the price of the home good is too low relative to the foreign good (foreign money supply is constant), the relative price of home goods decreases. Higher home consumption coupled with a lower relative price of home goods unambiguously increases the demand for home goods, raising short-run home output Y.[43] The decrease of the relative price of the home good leads – similar as in response to a home productivity shock – to a deterioration of the domestic terms of trade, benefiting foreign consumers and raising foreign consumption. Net exports increase and the home country runs a positive current account.[44] Foreign households again earn a positive return on their portfolio which is a second reason for the rise in their wealth. Overall, the wealth effect, which induces the foreign household to work less, is stronger than the substitution effect away from leisure to consumption, leading to a slight fall in foreign output and a marginal rise in the price of the foreign good. The increase in home money supply causes a nominal depreciation of the domestic currency, i.e., S rises. The depreciation is even larger than 1%, which would be the depreciation under flexible prices, due to the real effects of a change in money supply (see equation (83)).

[42] Take, e.g., the sum of future home stochastic discount factors $\sum_{i=0}^{\infty} \beta^i \hat{SDF}_{2+i}$. Setting $\beta = 1$, this term simplifies to $\sum_{i=0}^{\infty} \hat{SDF}_{2+i} = -\rho(\hat{C}_2 - \hat{C}_1) - \rho(\hat{C}_3 - \hat{C}_2) - ... - \rho(0 - \hat{C}_x)$, where the zero in the last term arises due to the fact that in this period the economy is back in the steady state and $\hat{C}_x$ is the last period before the economy is back in the steady state. This sum reduces to $-\rho(0 - \hat{C}_1) = \rho\hat{C}_1$, which is equal to the negative of the stochastic discount factor on impact $\hat{SDF}_1$.

[43] Home households work more since the substitution effect away from leisure to consumption is stronger than the wealth effect which would induce him to work less.

[44] While the current account is positive, again the change in net foreign assets ΔNFA is negative. This discrepancy reflects valuation effects resulting from portfolio diversification. Since the home country chooses the optimal portfolio $\tilde{\boldsymbol{\alpha}}^{\text{flex}}$, it suffers a capital loss on its portfolio when a home monetary shock realizes, i.e., $\tilde{\boldsymbol{\alpha}}\hat{\mathbf{r}}_{x,t} < 0$. This capital loss is larger than the positive movement in the trade account and therefore leads to a slight decrease in the home country's net foreign assets.

Before discussing the response of equity and bond prices to a home monetary shock in detail, we again assess the response of future consumption and future stochastic discount factors first. Impulse responses of selected variables, including consumption, are provided in Figure A.2 in Appendix G. Both the home and foreign stochastic discount factors decrease on impact due to the increase in home and foreign consumption, respectively. After impact, consumption is decreasing (no hump-shaped path) resulting in positive future stochastic discount factors, which positively affect asset prices.

Consider now the response of bond and equity prices to a home monetary shock. Focusing on the bond market first, both home and foreign nominal bond prices, Q_b and Q_b^*, increase when home money supply rises. On the one hand, households have higher incentives to invest in bonds on impact since consumption in the next period is expected to be lower (positive future stochastic discount factor). On the other hand, the anticipated rise in the future price level $(\pi_2, \pi_2^* > 0)$[45] lowers the bond's payoff and thereby has a negative effect on its price. On aggregate, the effect of the positive future stochastic discount factor is stronger, so that the net change in the bond's price is positive. The increase of the home relative to the foreign bond price is sharper since the home economy and hence, home consumption, is more heavily affected by the shock.

Focusing on the equity market, the nominal price of home equity Q_e rises on impact. Due to the decline in future consumption the home household has higher incentives to invest in home equity (positive future home stochastic discount factors). Expected future home dividends are (on average) lower than before the shock since the price of the home good is below the optimal level until it has fully adjusted (see Π in Figure A.2 in Appendix G), having a negative effect on the real home equity price. Since the effect of the positive future home stochastic discount factors is stronger, the net change of the real home equity price is positive. In addition, the increase in money supply yields home inflation on impact, so that home equity price increases even stronger in nominal than in real terms.

The nominal price of foreign equity Q_e^* rises on impact, too. In real terms, the price is positively affected by positive future foreign stochastic discount factors and the (on average) slightly higher future foreign dividends. However, the second effect is so small that it can be neglected. Thus, the foreign real equity price rises due to the anticipated decline in future foreign consumption. In nominal terms, the foreign equity price rises less than in real terms, since the decrease in the foreign consumer price level results in foreign deflation on impact.

Compared to the foreign equity price, the increase in the home equity price is much stronger for two reasons. First, home consumption is stronger affected by the shock (and hence, future home consumption is expected to be lower) and second, the home consumer price level increases while the foreign consumer price level decreases on impact.

The effects of a *foreign* monetary shock are symmetric to the ones of a home monetary shock just described.

[45] We have $\pi_2 > 0$ due to the hump-shaped response of P which in turn is a result of the staggered adjustment of prices in response to an increase in home money supply. $\pi_2^* > 0$ due to the non-hump-shaped response of P^*.

4.3 Fixed Exchange Rate Regime: Steady State Portfolio Allocation

In this and the next two subsections, we will discuss the case of a fixed instead of a flexible exchange rate regime. This subsection will first explain and derive the optimal portfolio allocation under such a fixed exchange rate regime, before subsections 4.4 and 4.5 will analyze how economic variables, in particular asset prices, respond to productivity and monetary shocks.

The parametrization is the same as presented in Table 2, with the exception of ϵ now going to infinity. The latter reflects that the foreign monetary authority adjusts foreign money supply to insulate the nominal exchange rate from the effects of shocks.

Under a fixed exchange rate regime, the optimal portfolio allocation will differ from the one that is optimal under the flexible regime. The reason for this is the just mentioned adjustment in foreign money supply in response to a shock under the fixed regime. This change in foreign money supply affects agent's income and consumption level, and also the real return on all assets. In other words, the home household's exposure to relative consumption risk and the assets' risk characteristics have changed.

To obtain the steady sate portfolio allocation under a fixed exchange rate regime, we proceed as in section 4.1 and investigate the degree to which each asset is useful in hedging against relative consumption risk resulting from the four shocks in the model. As before, we do this by looking at the (non-zero) covariance between relative marginal utilities and ex post excess returns when there is no trade in financial assets. Table 5 shows the response of relative marginal utilities and asset returns to productivity and monetary shocks on impact, given that agents hold the zero portfolio and the foreign country operates a unilateral peg.

	cd	$\hat{r}_{e_H}$	$\hat{r}_{e_F}$	$\hat{r}_{b_H}$	$\hat{r}_{b_F}$	$\hat{r}_{e_H} - \hat{r}_{e_F}$	$\hat{r}_{b_H} - \hat{r}_{b_F}$
u	0.069	0.685	0.259	0.234	0.234	0.426	0.000
u^*	-0.069	0.106	0.532	0.076	0.076	-0.426	0.000
m	0.000	0.520	0.520	-0.464	-0.464	0.000	0.000
m^*	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 5: Responses of relative marginal utilities and asset returns to shocks under a fixed exchange rate regime, given that agents hold the zero portfolio $\tilde{\boldsymbol{\alpha}}^0$. For returns the values represent the percentage deviation from the steady state in response to a shock of one standard deviation. For cd and excess returns within asset classes, $\hat{r}_{e_H} - \hat{r}_{e_F}$ and $\hat{r}_{b_H} - \hat{r}_{b_F}$, the values represent differences in percentage points.

In a first step, we focus on productivity shocks. The results suggest that, similar as under a flexible exchange rate regime, a positive *home* (*foreign*) productivity shock leads to an increase (decrease) in relative marginal utilities, an increase (increase) in all asset returns and a positive (negative) excess return on home over foreign equity. The excess return on home over foreign bonds, however, is zero, and therefore, under a fixed exchange rate regime, not affected by a productivity shock. This is due to the foreign monetary authority's intervention, which eliminates any unexpected exchange rate changes, i.e. any exchange rate risk. Therefore, the excess return on the home bond, which is equal to the negative of the change in the exchange rate and serves as a possibility to hedge against exchange rate risk, is zero, too. This implies that the home household is indifferent between any size of a bond portfolio consisting of a positive position in home bonds matched by the same negative position in foreign bonds. And it implies that the foreign monetary authority's intervention eliminates the effectiveness of nominal bonds as a hedging device, so that hedging against productivity shocks is only feasible through trading equity holdings. Thus, in contrast to the case of a flexible exchange rate regime, there exists only one portfolio allocation that allows hedging against both a home and a foreign productivity shock: a positive position in foreign equity that is matched by the same negative position in home equity.

In a next step, we focus on monetary shocks. The results suggest that, under a fixed exchange rate regime, a *home* monetary shock has no effect on relative marginal utilities. The reason is that, in response to such a shock, the foreign monetary authority adjusts foreign money supply by the same amount as home money changes. Therefore, both countries are identically exposed to changes in domestic and foreign money supply and hence, the change in home consumption equals the change in foreign consumption, the real exchange rate does not change, and cd is zero. Although the results in Table 5 explicitly assume that agents hold a zero portfolio, the finding that a home monetary shock has no effect on relative marginal utilities also holds for a non-zero portfolio. In such a case, the shock would affect returns on all four assets, but the response of asset returns would be identical within both types of assets, so that, for any symmetric portfolio allocation across countries, the change in the portfolio return will be the same across countries and may therefore affect absolute but not relative marginal utilities. To sum up, a home monetary shock only leads to aggregate but no country-specific risk. For this reason, the shock does not create incentives for trade in financial assets.

For a *foreign* monetary shock, the results suggest that such a shock has no effect on any of the variables under investigation. This is due to the foreign monetary authority's immediate intervention, so that money supply remains unchanged and there is no adjustment in the exchange rate.

Compared to a flexible exchange rate regime, agents in the two countries are thus only exposed to three instead of all four shocks. Furthermore, only two of them – home and foreign productivity shocks – generate differences in relative marginal utilities, against which the home household can hedge. As outlined, to hedge against these two risks, only equity holdings can be considered and the optimal diversification strategy for home households to avoid fluctuations in relative marginal utilities consists of a portfolio with a positive position in foreign equity matched by the negative equivalent in home equity. This portfolio can be combined with a positive position in one bond and an equivalent negative position in the other country's bond (although they provide no insurance).

The numerical result can be derived using equation (64): the optimal portfolio allocation under a fixed exchange rate regime is $\tilde{\boldsymbol{\alpha}}^{\text{fix}} = \begin{pmatrix} -1.49 & 1.49 & 0.76 \end{pmatrix}$. As mentioned before, these values reflect the home household's real external asset holdings. Real equity holdings of the home household, e_H and e_F, can be derived as outlined in subsection 4.1, which yields $e_H = 0.64$ and $e_F = 0.36$. Therefore, the fixed exchange rate regime does not imply a drastically different international portfolio allocation relative to the flexible regime. The home household holds 64% of home equity, which implies a slightly stronger bias towards home equity than under a flexible exchange rate regime. The home household trades slightly less equities (despite the fact that these are the only assets that can be used for hedging purposes). The reason for this is twofold. First, relative consumption risk is slightly lower under the fixed regime, so that less insurance is necessary. Second, the excess return on home over foreign equity is larger, implying that under the fixed regime, less equity trade is needed to hedge against the same consumption risk.

4.4 Fixed Exchange Rate Regime: Response to Shocks in the Home Country

This and the next subsection will analyze how economic variables, in particular asset prices, respond to shocks, given that the home household holds the optimal portfolio $\tilde{\boldsymbol{\alpha}}^{\text{fix}}$. The results are shown in Table 6.

In general, under a fixed exchange rate regime, a shock to either country is always associated with a change in foreign money supply, since foreign money supply instead of the exchange rate adjusts. Therefore, in addition to the impact of the original shock, countries are also affected by the change in foreign money supply. The intensity of the second effect depends on how strongly the foreign monetary authority has to adjust foreign money supply to keep the nominal exchange rate constant. A certain variable's response to a productivity or monetary shock will be approximately equal to its response under a flexible exchange rate regime plus the adjustment in foreign money supply times the response of this variable to a foreign monetary shock under the flexible regime.[46]

In the remainder of this subsection, we analyze the responses to shocks in the *home* country. Impulse responses of selected variables are provided in Figures A.3 and A.4 in Appendix G. As seen in section 4.2, without the intervention of the foreign monetary authority, a positive home productivity and a positive home monetary shock would both lead to an appreciation of the foreign currency (S increases). Under the fixed regime, however, the foreign monetary authority responds to both shocks by raising foreign money supply M^*, so that the resulting depreciation pressure on the foreign currency offsets the effect of the shock. The results in Table 6 suggest that this increase in foreign money supply is stronger in response to a home monetary than a home productivity shock (+0.1% versus +1.0%). Therefore, the difference in shock responses of economic variables across exchange rate regimes will always be larger for a home monetary than a home productivity shock. Since, under the flexible regime, the signs of the shock responses are the same across the two shocks for most of the variables, the effect of the intervention on this response is often similar, too, in the sense that it either amplifies or reduces the response of a variable to both shocks.

[46]Consider, e.g., the response of home output to a home productivity shock under the fixed regime. This effect is given by $0.368 + 0.1(-0.088) + e_t = 0.362$, where the first and third value are extracted from Table 4, the second value stems from Table 6 and e is an error term arising from the fact that agents do not hold the same steady state portfolio allocation across regimes and from general equilibrium adjustment effects. In this example $e = 0.003$.

	$\hat{Y}$	$\hat{Y}^*$	$\hat{C}$	$\hat{C}^*$	$\hat{CA}$	$\hat{\tau}$	$\hat{S}$	$\hat{Q}$
u	0.362	-0.016	0.223	0.123	0.040	0.398	0.000	0.199
u^*	-0.052	0.326	0.087	0.186	-0.040	-0.398	0.000	-0.199
m	0.536	0.536	0.536	0.536	0.000	0.000	0.000	0.000
m^*	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

	$\hat{M}$	$\hat{M}^*$	$\hat{P}$	$\hat{P}^*$	$\hat{P}_H$	$\hat{P}_F^*$	$\hat{\Pi}$	$\hat{\Pi}^*$
u	0.000	0.100	-0.223	-0.024	-0.322	0.076	5.727	-0.571
u^*	0.000	-0.100	-0.087	-0.286	0.013	-0.385	-0.062	6.236
m	1.000	1.000	0.464	0.464	0.464	0.464	-7.497	-7.497
m^*	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

	$\hat{Z}_e$	$\hat{Z}_e^*$	$\hat{Q}_e$	$\hat{Q}_e^*$	$\hat{Q}_b$	$\hat{Q}_b^*$
u	0.500	0.107	0.277	0.083	-0.085	-0.085
u^*	0.088	0.481	0.001	0.195	-0.037	-0.037
m	0.854	0.854	1.319	1.319	0.383	0.383
m^*	0.000	0.000	0.000	0.000	0.000	0.000

Table 6: Responses to shocks under a fixed exchange rate regime, given that the home agent holds $\tilde{\boldsymbol{\alpha}}^{\text{fix}}$. The values represent the percentage deviation from the steady state in response to a shock of one standard deviation ($\sigma_a = \sigma_m = 0.01$). If there is, e.g., a positive home productivity shock of one standard deviation, then Y_t rises by 0.36% on impact. An exception is the current account, $\hat{CA}$, which is defined to be the absolute change from the steady state as a fraction of the steady state output level.

Focusing on the response of home output to home shocks first, a home productivity or a home monetary shock alone yields an increase in home and a decrease in foreign output. However, since the foreign monetary authority reacts to these shocks by increasing foreign money supply (which has a decreasing effect on home output and an increasing effect on foreign output), the increase in home relative to foreign output is smaller than the one observed under a flexible exchange rate regime. Home output increases less in response to both shocks, while foreign output decreases to a smaller extent when a home productivity shock realizes, and even increases (by the same amount as the increase in home output) when a home monetary shock occurs.

In a next step, we focus on the response of home and foreign consumption to home shocks. Home and foreign consumption both increase in response to a positive home monetary or productivity shock alone. Furthermore, the intervention by the foreign monetary authority, has an additional positive effect on both home and foreign consumption, amplifying the response to both shocks. The amplification is stronger for foreign consumption since the foreign economy is more heavily affected by a change in foreign money supply. This stronger increase in consumption and the resulting changes in future consumption affect future stochastic discount factors. The response of the relevant stochastic discount factors to both shocks is amplified, too. As outlined earlier, the impulse responses of home and foreign consumption to a monetary shock are standard, while those to a productivity shock are hump-shaped. Compared to a flexible exchange rate regime, the response of next period's stochastic discount factor (which will be important when discussing the response of bond prices) to a home productivity shock is slightly more negative under a fixed exchange rate regime, since future consumption increases stronger.[47] In response to a home monetary shock, next period's stochastic discount factor is more positive under a fixed exchange rate regime, because future consumption decreases stronger.[48] The sums of future home and foreign stochastic discount factors (which will be important when discussing the response of equity prices) are larger in response to both shocks compared to the ones under a flexible exchange rate regime, due to the stronger response of consumption on impact, which yields a stronger decrease of future consumption.

In a next step, we discuss the response of asset prices to home shocks. Considering home productivity shocks first, we know from section 4.2, that under a flexible exchange rate regime, home and foreign nominal bond prices, Q_b and Q_b^*, decrease in response to such a shock, since the negative effect from a lower future stochastic discount factor more that offsets the positive effect from future deflation. Under a fixed exchange rate regime, this negative response of home and foreign bond prices is even stronger. Concerning the price of the home bond, fixing the exchange rate amplifies both, the decrease in the future home stochastic discount factor and future home deflation. In net terms, the first effect dominates slightly, so that, on aggregate, the price of the home bond decreases marginally stronger than under a flexible exchange rate regime. Concerning the price of the foreign bond, the response to a home productivity shock is more negative due to positive instead of negative future foreign inflation (see Figures A.1 and A.3 in Appendix G), and, to a more limited extent, due to the stronger decrease of the future foreign stochastic discount factor.

Turning to equity prices, Q_e and Q_e^*, we know that for both of them, the response to a positive home productivity shock under the flexible exchange rate regime is positive. The reason is that for both prices the positive effect from higher stochastic discount factors and higher future dividends more than offsets the negative effect from the decrease in the price level on impact. Under a fixed exchange rate regime, this positive response of home and foreign equity prices is amplified. Concerning the price of home equity, fixing the exchange rate yields even higher future home stochastic discount factors and a marginally lower increase in future home dividends. Therefore, the real home equity price Z_e increases more under a fixed regime. The decrease in the home price level on impact is stronger. Overall, the positive effect from the higher real equity price dominates the negative effect from the stronger decrease in the home price level, so that, in net terms, the nominal home equity price rises slightly stronger under the fixed regime. Concerning the price of foreign equity, fixing the exchange rate yields even higher future foreign stochastic discount factors and a lower increase in future foreign dividends. Since the second effect dominates, the real foreign equity price Z_e^* rises slightly less under a fixed

[47] When home and foreign consumption increase more on impact, the increase from the first to the second period is stronger, i.e., $C_2 - C_1$ and $C_2^* - C_1^*$ become larger, too. Then, the future stochastic discount factors $S\hat{D}F_2 = -\rho(\hat{C}_2 - \hat{C}_1) = -\rho(C_2 - C_1)/C$ and $S\hat{D}F_2^* = -\rho(C_2^* - C_1^*)/C^*$ are more negative.

[48] When home and foreign consumption increase more on impact, the fall from the first to the second period is stronger, i.e., $C_2 - C_1$ and $C_2^* - C_1^*$ are more negative. Then, the future stochastic discount factors $S\hat{D}F_2 = -\rho(C_2 - C_1)/C$ and $S\hat{D}F_2^* = -\rho(C_2^* - C_1^*)/C^*$ are larger.

regime. The decrease in the foreign price level on impact is weaker. Overall, the positive effect from lower foreign deflation dominates the negative effect from the lower real equity price, so that, in net terms, the nominal foreign equity price rises stronger under the fixed regime.

In sum, the intervention of the foreign monetary authority amplifies the effect of a home productivity shock on all nominal asset prices. The response of the home bond price (home equity price) is more negative (more positive) and driven (mainly driven) by the stronger response of the future home stochastic discount factor(s). The more negative response of the foreign bond price is mainly due to an increase instead of a decrease in the future foreign price level. The more positive response of the foreign equity price is driven by less deflation on impact and higher future foreign stochastic discount factors. A comparison between the two countries reveals that the fixed exchange rate regime has a stronger impact on foreign than on home asset prices. This is because an increase in foreign money supply affects more heavily the foreign economy, in particular foreign consumption and the foreign price level.

Now we turn to the response of asset prices to a home monetary shock. As mentioned earlier, in response to a positive home monetary shock, the foreign monetary authority increases its money supply one to one by the size of the increase in home money supply. Therefore, the home and foreign country are symmetrically affected by the shock and the shock responses of all variables are identical across countries (the ones of the current account, the terms of trade and the real exchange rate are zero). In this case, the response of asset prices to a home monetary shock under the fixed exchange rate regime simply equals the sum of the asset price response to (a) a home and (b) a foreign monetary shock under the flexible regime. The response of the home bond price to a positive home monetary shock, e.g., would then amount to $0.289 + 0.094 = 0.383$ (see Tables 4 and 6). All asset prices increase more than under a flexible regime since under a fixed regime, money supply increases not only in the home but also in the foreign country. The main driver behind this stronger increase is the same as the main driver behind the rise in asset prices in response to a *foreign* monetary shock under the flexible regime.

Focusing on bond prices, Q_b and Q_b^*, their more positive response under the fixed exchange rate regime is driven by a higher future stochastic discount factor. This can be inferred from the fact that the increase of home and foreign bond prices in response to a positive *foreign* monetary shock under the flexible regime is driven by the positive effect from a higher future stochastic discount factor.

Focusing on equity prices, the more positive increase in the home equity price Q_e under the fixed exchange rate regime is mainly driven by higher future home stochastic discount factors. This can be inferred from the fact that the increase in the home equity price in response to a positive *foreign* monetary shock under a flexible regime is driven by the positive effect from higher future home stochastic discount factors and slightly higher future home dividends (which are neglectable). The more positive response of the foreign equity price Q_e^* under a fixed exchange rate regime is driven by higher future stochastic discount factors and an increasing instead of a decreasing foreign price level on impact. This can be inferred from the fact that the increase in the foreign equity price in response to a positive *foreign* monetary shock under a flexible regime is driven by the positive effects from higher future foreign stochastic discount factors and a higher foreign price level on impact.

In sum, the intervention of the foreign monetary authority amplifies the effect of a home monetary shock on all nominal asset prices. The more positive response of prices of home bonds, foreign bonds and home equity is driven (or mainly driven in the case of the latter) by the higher future stochastic discount factor(s). The more positive response of the foreign equity price is driven by both the change in the foreign price level on impact and the higher future foreign stochastic discount factors. Furthermore, it is straightforward that foreign asset prices are more heavily affected by the increase in foreign money supply and therefore the amplification of their response is stronger.

4.5 Fixed Exchange Rate Regime: Response to Shocks in the Foreign Country

After having analyzed the responses to home shocks under a fixed exchange rate regime in the previous subsection, we do now the same for *foreign* shocks.

Focusing on a foreign monetary shock first, the results in Table 6 suggest that, under a fixed exchange rate regime, such a shock has no effect on any of the variables under investigation. This is due to the immediate intervention of the foreign monetary authority, so that money supply remains unchanged and there is no adjustment in the exchange rate. Therefore, a fixed exchange rate regime acts as a perfect shock absorber in case of a foreign monetary shock. As a consequence, the response of all asset prices to such a shock is weaker under a fixed than under a flexible exchange rate regime.

In a second step, we focus on the response to a (positive) foreign productivity shock. As seen in section 4.2, under a flexible exchange rate regime, such a shock leads to a depreciation of the foreign currency (S decreases). Under a fixed exchange rate regime, however, the foreign monetary authority offsets this depreciation by lowering foreign money supply M^*. Table 6 shows that, in response to a foreign productivity shock of one standard deviation, foreign money supply is lowered by 0.1%.

Assessing the response of asset prices to a positive foreign productivity shock, we know that, under a flexible exchange rate regime, bond prices decrease and equity prices increase. The shock responses were symmetric to the responses to a home productivity shock. Under the fixed regime, however, this symmetry does not apply anymore: as seen in the previous section, the response of all asset prices to a home productivity shock is amplified under a fixed exchange rate regime. For a foreign productivity shock, in contrast, the response of all asset prices is reduced. There are two reasons for this. First, the fixed regime involves an asymmetric intervention since it is operated as a one-sided peg. Second, this intervention differs across productivity shocks: while for a home productivity shock, foreign money supply is increased, it is decreased in the case of a foreign productivity shock. Since the sizes of these changes in money supply are the same in absolute terms, the magnitude of amplification in the case of a home productivity shock is the same as the magnitude of reduction in the case of a foreign productivity shock. The fixed regime contributes, e.g., 0.072 percentage points to the increase in the foreign equity price when a home productivity shock occurs and -0.072 percentage points when a foreign productivity shock realizes.

The reduction in the response of all asset prices to a foreign productivity shock under a fixed exchange rate regime implies that the response of bond prices is less negative and the one of equity prices is less positive. The drivers are the same as the ones behind the amplification of the response to a home productivity shock, although the effects go in opposite directions. The response of the home bond price (home equity price) is less negative (less positive) and driven (mainly driven) by the weaker response of the future home stochastic discount factor(s). The less negative response of the foreign bond price is mainly due to higher future deflation, while the less positive response of the foreign equity price is driven by higher deflation on impact and lower future foreign stochastic discount factors. A comparison between the two countries reveals that the fixed exchange rate regime has a stronger impact on foreign asset prices than on home asset prices. This is because a decrease in foreign money supply affects more heavily the foreign economy, in particular foreign consumption and the foreign price level.

4.6 Summary and Policy Implications

In sum, three main findings emerge from my analysis. First, asset price responses to economic shocks differ across exchange rate regimes. Asset prices are more responsive to shocks in the home and less responsive to shocks in the foreign country when the foreign country operates a unilateral peg instead of having a flexible exchange rate regime. Second, while the differences in home asset price responses to shocks tend to be small, the differences in foreign asset price responses are more pronounced. Third, the differences are stronger for nominal relative to real shocks.

Under a peg, the foreign country insulates the nominal exchange rate from the effects of shocks by adjusting money supply. Hence, when a country-specific shock occurs foreign money supply instead of the exchange rate adjusts.

Consider first the response of home asset prices to shocks in the home country. In response to both a positive home monetary and a positive home productivity shock, the foreign monetary authority raises foreign money supply to avoid the appreciation of its currency. This expansionary policy has real consequences in the short run when prices cannot fully adjust. It leads to a decrease in the relative price of foreign goods and an improvement of the home terms of trade (τ increases less). Home consumption rises sharper on impact, affecting the home agents' intertemporal consumption-savings decision. The response of future home stochastic discount factors is amplified, which in most cases implies a stronger increase of the demand for home assets. An exception is the sharper fall of the demand for the home bond when a productivity shock realizes. As a consequence, the response of home asset prices under the fixed regime is amplified. In terms of magnitude, however, the amplification is small. The contribution of the fixed regime to asset price movements is stronger when a nominal as opposed to a real shock occurs since in response to the former foreign monetary policy is more expansionary.

Foreign asset prices also move more to shocks in the home country under a unilateral peg.

In the foreign country, the increase in foreign money supply leads to a rise in foreign money balances and thus to a stronger rise in foreign consumption, amplifying the response of future foreign stochastic discount factors. This amplification in combination with the increase in the foreign price level are the drivers behind the stronger response of foreign asset prices. The effect of the fixed regime on the response of foreign relative to home asset prices is more pronounced. Again the effect of the regime is even stronger when a nominal as opposed to a real shock occurs.

In contrast to previous findings, home asset prices respond less to shocks in the foreign country when the foreign country operates a unilateral peg instead of having a floating exchange rate regime. Clearly, and as mentioned above, home asset prices are less responsive to a foreign money supply shock. But the response to a foreign productivity shock is weaker, too. The reason for this is that the foreign monetary authority lowers foreign money supply, which reduces the effect of a foreign positive productivity shock on home consumption and hence reduces the response of future home stochastic discount factors. Foreign asset prices respond less to foreign shocks, too.

As outlined earlier, changes in the steady state portfolio allocation could also be a possible reason for changes in the response of economic variables to shocks. However, since a fixed exchange rate regime does not imply a drastically different international portfolio allocation than a flexible regime, the different response of economic variables across regimes is only weakly influenced by changes in the portfolio allocation.

For the US and China, these results imply that under the prevailing unilateral peg in China, prices of US bonds and stocks move more when a shock in the US occurs relative to the response under a flexible USD/CNY exchange rate, but the effects are rather small. China relaxes the domestic monetary conditions in response to a positive US productivity or monetary shock to avoid an appreciation of the Renminbi. This leads to an improvement in the US terms of trade (relative to the terms of trade under a flexible USD/CNY exchange rate) and increases wealth in the US. Since the effect is only temporary (in the long run money is neutral), investors' demand for US stocks increases, pushing up prices of these assets. The demand for home bonds and the prices of such bonds move more, too.

Some economists argued that before the financial crisis monetary policy had been too expansionary fuelling the US asset price bubble, see e.g., Taylor (2007). My findings suggest that the effect of the Fed's expansionary monetary policy (captured by a positive value of m) on US asset prices was amplified by China's prevailing unilateral peg, but only to a limited extent.

5 Conclusion

Asset price movements have not only gained importance with the increasing scale of gross foreign asset and liability positions and the resulting global interdependences over the past decades, but are also affected by the increasing cross-border capital flows. Caballero and Krishnamurthy (2009) and Bernanke et al. (2011) argued that in the years before the outburst of the financial crisis, large capital inflows into US safe assets had not only contributed to the increase in safe asset prices but had also substantially pushed up prices of more risky US assets (stocks, real estate, including mortgage-backed securities). A large part of these capital inflows came from Asian emerging economies, such as China, who operate a one-sided peg against the US dollar. Given the potential contribution of large inflows from China to the rise in US safe and risky asset prices before the crisis, China's prevailing unilateral peg might have played a role for US asset price inflation.

Motivated by these developments, the purpose of this paper is to assess the contribution of the Chinese exchange rate regime to US asset price inflation before the outburst of the financial crisis and to analyze whether the nominal exchange rate regime (flexible versus unilateral peg) plays a role in the international transmission of shocks to financial markets in general. A similar exercise was done by Dellas and Tavlas (2013). However, a shortcoming of their model is that the portfolio allocation – which is of importance when analyzing asset prices – is ad hoc. Therefore, their model abstracts from endogenous portfolio choice. In this paper I overcome this shortcoming by making use of a newer method for analyzing portfolio choice in DSGE models proposed in Devereux and Sutherland (2011). Although the model does not replicate the portfolio structure of the US and China – China's net international investment position, e.g., consists of a large share of reserve assets – and has not been calibrated to these two countries, it provides nonetheless a simple framework that sheds light on the effects of alternative exchange rate regimes on asset prices.

Similar to the findings in Dellas and Tavlas (2013), my findings suggest that in general, asset price responses to shocks differ across exchange rate regimes. While the differences in the shock responses of home asset prices tend to be small, the differences in the responses of foreign asset prices are more pronounced. Applied to the US and China, US bond and stock prices are more responsive to domestic shocks under China's prevailing unilateral peg than under a floating USD/CNY exchange rate, but the extent of this amplification is limited. The reason for this stronger response of US asset prices under the peg is that, in response to a positive US monetary or productivity shock, China relaxes domestic monetary conditions to prevent an appreciation of the Renminbi. Given that prices are sticky in the short run, this slightly amplifies the shock response of US consumption and therefore also of US asset prices. Thus, under the prevailing unilateral peg, the Fed's expansionary monetary policy before the financial crisis resulted in a slightly but not substantially stronger US asset price inflation relative to the one that would have been observed under a floating USD/CNY exchange rate.

Several extensions could be interesting for future research. While the model in this paper was kept simple and only incorporated a small number of shocks (which implies only a few sources of uncertainty against which hedging is necessary), it would be interesting to look at a higher number and other types of shocks. For instance, one idea would be to follow the lines of Dellas and Tavlas (2013) and investigate the response of asset prices to a relocation of labor in China towards the traded sector or to US debt expansion. Another possibility would be to look at financial shocks, e.g., exogenous portfolio shifts or deviations from the covered interest rate parity. Such an inclusion of additional shocks would automatically result in incompleteness of financial markets, which would be an interesting extension given the extensive evidence of incompleteness in international financial markets.

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A Equations of the Foreign Country

Consumption index:

$$C_t^* = \left(\mu^{\frac{1}{\theta}} C_{F,t}^{*\frac{\theta-1}{\theta}} + (1-\mu)^{\frac{1}{\theta}} C_{H,t}^{*\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \tag{95}$$

Aggregate price index:

$$P_t^* = \left(\mu P_{F,t}^{*1-\theta} + (1-\mu)(P_{H,t}/S_t)^{1-\theta}\right)^{\frac{1}{1-\theta}} \tag{96}$$

Budget constraint:

$$P_t^* C_t^* + P_t^* NFA_t^* = P_{F,t}^* Y_t^* + P_t^* \sum_{k \in K'} \frac{1}{Q_t} \alpha_{k,t-1}^* r_{xk,t} + P_t^* NFA_{t-1}^* \frac{Q_{t-1}}{Q_t} r_{b_F,t} \tag{97}$$

where

$$\alpha_{e_{H,t}}^* \equiv e_{Ht}^* Z_{e,t} \tag{98}$$

$$\alpha_{e_{F,t}}^* \equiv (e_{Ft}^* - 1) Q_t Z_{e,t}^* \tag{99}$$

$$\alpha_{b_{H,t}}^* \equiv b_{h,t}^* Z_{b,t} \tag{100}$$

$$\alpha_{b_{F,t}}^* \equiv b_{F,t}^* Q_t Z_{b,t}^* \tag{101}$$

Optimal consumption-leisure tradeoff for foreign households:

$$\frac{L_t^{*\chi}}{C_t^{*-\rho}} = \frac{w_t^*}{P_t^*} \tag{102}$$

Optimal consumption-savings decision and portfolio choices, i.e., the first order conditions w.r.t. NFA_t^*, $\alpha_{e_H,t}^*$, $\alpha_{e_F,t}^*$ and $\alpha_{b_H,t}^*$:

$$C_t^{*-\rho} = \beta E_t \left[C_{t+1}^{*-\rho} r_{b_F,t+1} \frac{Q_t}{Q_{t+1}} \right] \tag{103}$$

$$E_t \left[C_{t+1}^{*-\rho} \left(\frac{r_{e_H,t+1} - r_{b_F,t+1}}{Q_{t+1}} \right) \right] = 0 \tag{104}$$

$$E_t \left[C_{t+1}^{*-\rho} \left(\frac{r_{e_F,t+1} - r_{b_F,t+1}}{Q_{t+1}} \right) \right] = 0 \tag{105}$$

$$E_t \left[C_{t+1}^{-\rho} \left(\frac{r_{b_H,t+1} - r_{b_F,t+1}}{Q_{t+1}} \right) \right] = 0 \tag{106}$$

Production function:

$$Y_t^*(j) = A_t^* L_t^*(j) \tag{107}$$

Productivity shock:

$$\log A_t^* = \psi_a \log A_{t-1}^* + u_t^* \tag{108}$$

Evolution of the price for foreign firms' goods:

$$P_{F,t}^* = [(1-\kappa)\tilde{P}_{F,t}^{*1-\phi} + \kappa P_{F,t-1}^{*1-\phi}]^{\frac{1}{1-\phi}} \tag{109}$$

Newly set price:

$$\tilde{P}_{F,t}^* = \mathcal{M}\frac{E_t \sum_{i=0}^{\infty} \Omega_{t+i}^* \kappa^i \frac{w_{t+i}^*}{A_{t+i}^*} X_{F,t+i}^*}{E_t \sum_{i=0}^{\infty} \Omega_{t+i}^* \kappa^i X_{F,t+i}^*} \tag{110}$$

Money demand equation:

$$M_t^* v = P_t^* C_t^* \tag{111}$$

B Budget Constraint

We can rewrite the domestic household's budget constraint, i.e. equation (5), in terms of real asset prices ($Z_{e,t} = Q_{e,t}/P_t$, $Z_{e,t}^* = Q_{e,t}^*/P_t^*$, $Z_{b,t} = Q_{b,t}/P_t$ and $Z_{b,t}^* = Q_{b,t}^*/P_t^*$):

$$\begin{aligned} &P_t C_t + e_{H,t} Z_{e,t} P_t + e_{F,t} Z_{e,t}^* P_t^* S_t + b_{H,t} Z_{b,t} P_t + b_{F,t} Z_{b,t}^* P_t^* S_t = w_t L_t + e_{H,t-1}(Z_{e,t} P_t + \Pi_t P_t) \\ &+ e_{F,t-1} S_t (Z_{e,t}^* P_t^* + \Pi_t^* P_t^*) + b_{H,t-1} R_t Z_{b,t-1} P_{t-1} + b_{F,t-1} R_t^* Z_{b,t-1}^* P_{t-1}^* S_t \end{aligned} \tag{112}$$

Using the definitions of $\alpha_{e_H,t}$, $\alpha_{e_F,t}$, $\alpha_{b_H,t}$ and $\alpha_{b_F,t}$ (see equations (6)-(9)), we can rewrite this equation as follows

$$\begin{aligned} &P_t C_t + P_t(\alpha_{e_H,t} + \alpha_{e_F,t} + \alpha_{b_H,t} + \alpha_{b_F,t}) + Z_{e,t} P_t = w_t L_t + \alpha_{e_H,t-1}\frac{Z_{e,t} + \Pi_t}{Z_{e,t-1}} P_t + (Z_{e,t} + \Pi_t) P_t \\ &+ \alpha_{e_F,t-1}\frac{Z_{e,t}^* + \Pi_t^*}{Z_{e,t-1}^*}\frac{Q_t}{Q_{t-1}} P_t + \alpha_{b_H,t-1} R_t \frac{P_{t-1}}{P_t} P_t + \alpha_{b_F,t-1} R_t^* \frac{P_{t-1}^*}{P_t^*}\frac{Q_t}{Q_{t-1}} P_t \end{aligned} \tag{113}$$

Using the definitions of real returns and the fact that $R_t = r_{b_H,t}\frac{P_t}{P_{t-1}}$ and $R_t^* = r_{b_F,t}\frac{Q_{t-1}}{Q_t}\frac{P_t^*}{P_{t-1}^*}$ we have

$$P_t C_t + P_t \sum_{k\in K} \alpha_{k,t} = w_t L_t + P_t \Pi_t + P_t \sum_{k\in K} \alpha_{k,t-1} r_{k,t} \tag{114}$$

$$\Leftrightarrow P_t C_t + P_t NFA_t = w_t L_t + P_t \Pi_t + P_t \sum_{k\in K} \alpha_{k,t-1} r_{k,t} \tag{115}$$

which is exactly equation (10).

C Symmetric Non-Stochastic Steady State

$$r_{e_H} = r_{e_F} = r_{b_H} = r_{b_F} = \frac{1}{\beta} \tag{116}$$

$$NFA = 0 \tag{117}$$

$$L = L^* = \left(\frac{A^{1-\rho}}{\mathcal{M}}\right)^{\frac{1}{\rho+\chi}} \tag{118}$$

$$C = C^* = Y = Y^* = AL = \left(\frac{A^{1+\chi}}{\mathcal{M}}\right)^{\frac{1}{\rho+\chi}} \tag{119}$$

$$\Pi = \Pi^* = \left(\frac{\mathcal{M}-1}{\mathcal{M}}\right) Y \tag{120}$$

$$P = P_H = \tilde{P}_H = P^* = P_F^* = \tilde{P}_F^* = \frac{vM}{Y} \tag{121}$$

$$w = w^* = \frac{A}{\mathcal{M}} P \tag{122}$$

$$S = Q = \tau = 1 \tag{123}$$

$$Z_b = Z_b^* = \frac{\beta}{P} = \frac{\beta Y}{vM} \tag{124}$$

$$Z_e = Z_e^* = \frac{\Pi}{r_e - 1} = \frac{\beta}{1-\beta} \frac{\mathcal{M}-1}{\mathcal{M}} Y \tag{125}$$

Note that in the steady state I set $M = M^* = 1$ and $A = A^* = 10$. This, in combination with the values above, gives us the steady state values of 32 of the 35 variables. To find the remaining three variables, α_{e_H}, α_{e_F} and α_{b_H}, the Devereux-Sutherland algorithm is applied.

D First Order Approximations of the Equilibrium Equations

This section outlines the first order approximations of the model equations. If nothing else is stated, a hat indicates the log deviations of a variable from its steady state value and a variable without a time subscript denotes the steady state value.

- Consumer price indices:

$$\hat{P}_t = \mu \left(\frac{P_H}{P}\right)^{1-\theta} \hat{P}_{H,t} + (1-\mu) \left(\frac{P_F^* S}{P}\right)^{1-\theta} (\hat{P}_{F,t}^* + \hat{S}_t) \tag{126}$$

$$\hat{P}_t^* = \mu \left(\frac{P_F^*}{P^*}\right)^{1-\theta} \hat{P}_{F,t}^* + (1-\mu) \left(\frac{P_H}{SP^*}\right)^{1-\theta} (\hat{P}_{H,t} - \hat{S}_t) \tag{127}$$

Using the steady state values and equation (181) below, these equations simplify to

$$\hat{P}_t = \mu \hat{P}_{H,t} + (1-\mu)(\hat{P}_{F,t}^* + \hat{S}_t) \tag{128}$$

$$\Leftrightarrow \hat{P}_t = \hat{P}_{H,t} + (1-\mu)\hat{\tau}_t \tag{129}$$

and

$$\hat{P}_t^* = \mu \hat{P}_{F,t}^* + (1-\mu)(\hat{P}_{H,t} - \hat{S}_t) \tag{130}$$
$$\Leftrightarrow \hat{P}_t^* = \hat{P}_{F,t}^* - (1-\mu)\hat{\tau}_t \tag{131}$$

- Domestic household's budget constraint (17):

$$\begin{aligned} PC(\hat{P}_t + \hat{C}_t) + PNFA\hat{P}_t + P(NFA_t - NFA) = P_H Y(\hat{P}_{H,t} + \hat{Y}_t) \\ + P\sum_{k\in K'} \alpha_k r_{xk}(\hat{P}_t + \hat{\alpha}_{k,t-1}) + P\sum_{k\in K'} \alpha_k r(\hat{r}_{k,t} - \hat{r}_{b_F,t}) + PNFAr_{b_F}(\hat{P}_t + \hat{r}_{b_F,t}) \\ + Pr_{b_F}(NFA_{t-1} - NFA) \end{aligned} \tag{132}$$

Note that in the steady state all returns are equal, i.e., $r_{e_H} = r_{e_F} = r_{b_H} = r_{b_F} = r = \frac{1}{\beta}$ and hence, $r_{xk} = 0 \quad \forall k \in K'$. Therefore, and since $NFA = 0$, the second, fifth and seventh term are equal to zero. Define $N\hat{F}A_t = \frac{NFA_t}{Y}$, $\boldsymbol{\alpha} = \begin{pmatrix} \alpha_{e_H} & \alpha_{e_F} & \alpha_{b_H} \end{pmatrix}$ and $\hat{\mathbf{r}}'_{x,t} = \begin{pmatrix} \hat{r}_{e_H,t} - \hat{r}_{b_F,t} & \hat{r}_{e_F,t} - \hat{r}_{b_F,t} & \hat{r}_{b_H,t} - \hat{r}_{b_F,t} \end{pmatrix}$. We can rewrite the above equation as follows

$$PC(\hat{P}_t + \hat{C}_t) + PNFA_t = P_H Y(\hat{P}_{H,t} + \hat{Y}_t) + P\frac{1}{\beta}\sum_{k\in K'} \alpha_k \hat{r}_{xk,t} + P\frac{1}{\beta}NFA_{t-1} \tag{133}$$
$$\Leftrightarrow \hat{P}_t + \hat{C}_t + \frac{1}{Y}NFA_t = \hat{P}_{H,t} + \hat{Y}_t + \frac{1}{\beta Y}\boldsymbol{\alpha}\hat{\mathbf{r}}_{x,t} + \frac{1}{\beta Y}NFA_{t-1} \tag{134}$$
$$\Leftrightarrow N\hat{F}A_t = \frac{1}{\beta}N\hat{F}A_{t-1} + \hat{Y}_t - \hat{C}_t + \hat{P}_{H,t} - \hat{P}_t + \tilde{\boldsymbol{\alpha}}\hat{\mathbf{r}}_{x,t} \tag{135}$$

which is identical to equation (56).

Note that the change in net foreign assets is

$$\Delta N\hat{F}A_t = \frac{1-\beta}{\beta}N\hat{F}A_{t-1} + \hat{Y}_t - \hat{C}_t + \hat{P}_{H,t} - \hat{P}_t + \tilde{\boldsymbol{\alpha}}\hat{\mathbf{r}}_{x,t} \tag{136}$$

A first order approximation of the current account yields:

$$CA_t = \frac{P_H}{P}Y(\hat{P}_{H,t} - \hat{P}_t + \hat{Y}_t) - C\hat{C}_t + NFA_{t-1}(r_{b_F} - 1) \tag{137}$$
$$\Leftrightarrow \hat{CA}_t = \hat{P}_{H,t} - \hat{P}_t + \hat{Y}_t - \hat{C}_t + N\hat{F}A_{t-1}\left(\frac{1-\beta}{\beta}\right) \tag{138}$$

where $\hat{CA}_t = CA_t/Y$. The change in net foreign assets can therefore be rewritten as follows

$$\Delta N\hat{F}A_t = \hat{CA}_t + \tilde{\boldsymbol{\alpha}}\hat{\mathbf{r}}_{x,t} \tag{139}$$

where $\tilde{\boldsymbol{\alpha}}\hat{\mathbf{r}}_{x,t}$ are valuation effects.

- Gross real rates of return on home and foreign equity:

$$\hat{r}_{e_H,t+1} = (1-\beta)\hat{\Pi}_{t+1} + \beta\hat{Z}_{e,t+1} - \hat{Z}_{et} \tag{140}$$
$$\hat{r}_{e_F,t+1} = (1-\beta)\hat{\Pi}^*_{t+1} + \beta\hat{Z}^*_{e,t+1} - \hat{Z}^*_{et} + \hat{Q}_{t+1} - \hat{Q}_t \tag{141}$$

- Gross real rates of return on home and foreign bonds:

$$\hat{r}_{b_H,t+1} = -\hat{P}_{t+1} - \hat{Z}_{bt} \tag{142}$$
$$\hat{r}_{b_F,t+1} = -\hat{P}^*_{t+1} - \hat{Z}^*_{bt} + \hat{Q}_{t+1} - \hat{Q}_t \tag{143}$$

- Optimal consumption-leisure tradeoffs:

$$\rho\hat{C}_t + \chi\hat{L}_t = \hat{w}_t - \hat{P}_t \tag{144}$$
$$\rho\hat{C}^*_t + \chi\hat{L}^*_t = \hat{w}^*_t - \hat{P}^*_t \tag{145}$$

- Optimal consumption and portfolio choices of the home household:

$$\rho(E_t\hat{C}_{t+1} - \hat{C}_t) = E_t(\hat{r}_{b_F,t+1}) \tag{146}$$
$$E_t\hat{r}_{e_H,t+1} = E_t\hat{r}_{b_F,t+1} \tag{147}$$
$$E_t\hat{r}_{e_F,t+1} = E_t\hat{r}_{b_F,t+1} \tag{148}$$
$$E_t\hat{r}_{b_H,t+1} = E_t\hat{r}_{b_F,t+1} \tag{149}$$

- Optimal consumption and portfolio choices of the foreign household:

$$\rho(E_t\hat{C}^*_{t+1} - \hat{C}^*_t) + E_t\hat{Q}_{t+1} - \hat{Q}_t = E_t(\hat{r}_{b_F,t+1}) \tag{150}$$
$$E_t\hat{r}_{e_H,t+1} = E_t\hat{r}_{b_F,t+1} \tag{151}$$
$$E_t\hat{r}_{e_F,t+1} = E_t\hat{r}_{b_F,t+1} \tag{152}$$
$$E_t\hat{r}_{b_H,t+1} = E_t\hat{r}_{b_F,t+1} \tag{153}$$

- Production of the differentiated product:

$$\hat{Y}_t = \hat{A}_t + \hat{L}_t \tag{154}$$
$$\hat{Y}^*_t = \hat{A}^*_t + \hat{L}^*_t \tag{155}$$

- Productivity shocks:

$$\hat{A}_t = \psi_a\hat{A}_{t-1} + u_t \tag{156}$$
$$\hat{A}^*_t = \psi_a\hat{A}^*_{t-1} + u^*_t \tag{157}$$

- Newly set prices:

$$\hat{\tilde{P}}_{H,t} = \beta\kappa E_t \hat{\tilde{P}}_{H,t+1} + (1-\beta\kappa)(\hat{w}_t - \hat{A}_t) \tag{158}$$
$$\hat{\tilde{P}}^*_{F,t} = \beta\kappa E_t \hat{\tilde{P}}^*_{F,t+1} + (1-\beta\kappa)(\hat{w}^*_t - \hat{A}^*_t) \tag{159}$$

- Producer price indices:

$$\hat{P}_{H,t} = P_H^{-(1-\phi)}[(1-\kappa)\tilde{P}_H^{1-\phi}\hat{\tilde{P}}_{H,t} + \kappa P_H^{1-\phi}\hat{P}_{H,t-1}] \tag{160}$$
$$\hat{P}^*_{F,t} = P_F^{*-(1-\phi)}[(1-\kappa)\tilde{P}_F^{*1-\phi}\hat{\tilde{P}}^*_{F,t} + \kappa P_F^{*1-\phi}\hat{P}^*_{F,t-1}] \tag{161}$$

Using the steady state values, these equations simplify to

$$\hat{P}_{H,t} = (1-\kappa)\hat{\tilde{P}}_{H,t} + \kappa\hat{P}_{H,t-1} \tag{162}$$
$$\hat{P}^*_{F,t} = (1-\kappa)\hat{\tilde{P}}^*_{F,t} + \kappa\hat{P}^*_{F,t-1} \tag{163}$$

We can rewrite equation (162) as follows

$$\hat{P}_{H,t} - \beta\kappa E_t\hat{P}_{H,t+1} = (1-\kappa)(\hat{\tilde{P}}_{H,t} - \beta\kappa E_t\hat{\tilde{P}}_{H,t+1}) + \kappa(\hat{P}_{H,t-1} - \beta\kappa\hat{P}_{H,t}) \tag{164}$$
$$\Leftrightarrow \hat{\tilde{P}}_{H,t} - \beta\kappa E_t\hat{\tilde{P}}_{H,t+1} = \frac{1}{1-\kappa}(\hat{P}_{H,t} - \beta\kappa E_t\hat{P}_{H,t+1}) - \frac{\kappa}{1-\kappa}(\hat{P}_{H,t-1} - \beta\kappa\hat{P}_{H,t}) \tag{165}$$

Combining this equation with equation (158) yields

$$(1-\beta\kappa)(\hat{w}_t - \hat{A}_t) = \frac{1}{1-\kappa}(\hat{P}_{H,t} - \beta\kappa E_t\hat{P}_{H,t+1}) - \frac{\kappa}{1-\kappa}(\hat{P}_{H,t-1} - \beta\kappa\hat{P}_{H,t}) \tag{166}$$
$$\Leftrightarrow (1-\kappa)(1-\beta\kappa)(\hat{w}_t - \hat{A}_t) = (\hat{P}_{H,t} - \beta\kappa E_t\hat{P}_{H,t+1}) - \kappa(\hat{P}_{H,t-1} - \beta\kappa\hat{P}_{H,t}) \tag{167}$$
$$\Leftrightarrow \hat{P}_{H,t} - \hat{P}_{H,t-1} = \frac{(1-\kappa)(1-\beta\kappa)}{\kappa}(\hat{w}_t - \hat{A}_t - \hat{P}_{H,t}) + \beta(E_t\hat{P}_{H,t+1} - \hat{P}_{H,t}) \tag{168}$$

I define $\lambda = \frac{\kappa}{(1-\kappa)(1-\beta\kappa)}$ and use (144) to find

$$\hat{P}_{H,t} - \hat{P}_{H,t-1} = \frac{1}{\lambda}(\rho\hat{C}_t + \hat{P}_t + \chi\hat{L}_t - \hat{P}_{H,t} - \hat{A}_t) + \beta(E_t\hat{P}_{H,t+1} - \hat{P}_{H,t}) \tag{169}$$
$$\Leftrightarrow \pi_{H,t} = \frac{1}{\lambda}(\rho\hat{C}_t + \hat{P}_t + \chi\hat{L}_t - \hat{P}_{H,t} - \hat{A}_t) + \beta E_t\pi_{H,t+1} \tag{170}$$

where home producer price inflation $\pi_{H,t}$ is defined as $\pi_{H,t} = \hat{P}_{H,t} - \hat{P}_{H,t-1}$. Equivalently, we have

$$\hat{P}^*_{F,t} - \hat{P}^*_{F,t-1} = \frac{1}{\lambda}(\rho\hat{C}^*_t + \hat{P}^*_t + \chi\hat{L}^*_t - \hat{P}^*_{F,t} - \hat{A}^*_t) + \beta(E_t\hat{P}^*_{F,t+1} - \hat{P}^*_{F,t}) \tag{171}$$
$$\Leftrightarrow \pi^*_{F,t} = \frac{1}{\lambda}(\rho\hat{C}^*_t + \hat{P}^*_t + \chi\hat{L}^*_t - \hat{P}^*_{F,t} - \hat{A}^*_t) + \beta E_t\pi^*_{F,t+1} \tag{172}$$

where foreign producer price inflation $\pi^*_{F,t}$ is defined as $\pi^*_{F,t} = \hat{P}^*_{F,t} - \hat{P}^*_{F,t-1}$.

- Profit functions:

$$\hat{\Pi}_t = \frac{P_H Y}{P\Pi}(\hat{P}_{H,t} + \hat{Y}_t - \hat{P}_t) - \frac{wL}{P\Pi}(\hat{w}_t + \hat{L}_t - \hat{P}_t) \tag{173}$$

$$\hat{\Pi}^*_t = \frac{P^*_F Y^*}{P^*\Pi^*}(\hat{P}^*_{F,t} + \hat{Y}^*_t - \hat{P}^*_t) - \frac{w^* L^*}{P^*\Pi^*}(\hat{w}^*_t + \hat{L}^*_t - \hat{P}^*_t) \tag{174}$$

Using the steady state values we have

$$\hat{\Pi}_t = \frac{\mathcal{M}}{\mathcal{M}-1}(\hat{P}_{H,t} + \hat{Y}_t - \hat{P}_t) - \frac{1}{\mathcal{M}-1}(\hat{w}_t + \hat{L}_t - \hat{P}_t) \tag{175}$$

$$\hat{\Pi}^*_t = \frac{\mathcal{M}}{\mathcal{M}-1}(\hat{P}^*_{F,t} + \hat{Y}^*_t - \hat{P}^*_t) - \frac{1}{\mathcal{M}-1}(\hat{w}^*_t + \hat{L}^*_t - \hat{P}^*_t) \tag{176}$$

- Money demand equations:

$$\hat{M}_t - \hat{P}_t = \hat{C}_t \tag{177}$$

$$\hat{M}^*_t - \hat{P}^*_t = \hat{C}^*_t \tag{178}$$

- Monetary rules:

$$\hat{M}_t = \psi_m \hat{M}_{t-1} + m_t \tag{179}$$

$$\hat{M}^*_t = \psi_m \hat{M}^*_{t-1} - \epsilon \hat{S}_t + m^*_t \tag{180}$$

- Terms of trade:

$$\hat{\tau}_t = \hat{P}^*_{F,t} + \hat{S}_t - \hat{P}_{H,t} \tag{181}$$

- Real exchange rate:

$$\hat{Q}_t = \hat{P}^*_t + \hat{S}_t - \hat{P}_t \tag{182}$$

Note that we can rewrite $\hat{Q}_t$ using equation (181) to replace $\hat{S}_t$ and equations (129) and (131) to replace $\hat{P}_t$ and $\hat{P}^*_t$:

$$\hat{Q}_t = \hat{P}^*_t + \hat{S}_t - \hat{P}_t \tag{183}$$

$$\Leftrightarrow \hat{Q}_t = (\hat{P}^*_{F,t} - (1-\mu)\hat{\tau}_t) + (\hat{\tau}_t - \hat{P}^*_{F,t} + \hat{P}_{H,t}) - (\hat{P}_{H,t} + (1-\mu)\hat{\tau}_t) \tag{184}$$

$$\Leftrightarrow \hat{Q}_t = -2(1-\mu)\hat{\tau}_t + \hat{\tau}_t \tag{185}$$

$$\Leftrightarrow \hat{Q}_t = (2\mu - 1)\hat{\tau}_t \tag{186}$$

- Home goods market clearing:

$$Y\hat{Y}_t = \mu\left(\frac{P_H}{P}\right)^{-\theta} C[\hat{C}_t - \theta(\hat{P}_{H,t} - \hat{P}_t)] + (1-\mu)\left(\frac{P_H}{SP^*}\right)^{-\theta} C^*[\hat{C}^*_t - \theta(\hat{P}_{H,t} - \hat{S}_t - \hat{P}^*_t)] \tag{187}$$

Using the steady state values we have

$$\hat{Y}_t = \mu\hat{C}_t + (1-\mu)\hat{C}^*_t - \theta\mu(\hat{P}_{H,t} - \hat{P}_t) - \theta(1-\mu)(\hat{P}_{H,t} - \hat{S}_t - \hat{P}^*_t) \tag{188}$$

- Foreign goods market clearing:

$$Y^*\hat{Y}^*_t = \mu\left(\frac{P^*_F}{P^*}\right)^{-\theta} C^*[\hat{C}^*_t - \theta(\hat{P}^*_{F,t} - \hat{P}^*_t)] + (1-\mu)\left(\frac{P^*_F S}{P}\right)^{-\theta} C[\hat{C}_t - \theta(\hat{P}^*_{F,t} + \hat{S}_t - \hat{P}_t)] \tag{189}$$

Using the steady state values and (181) we have

$$\hat{Y}^*_t = \mu[\hat{C}^*_t - \theta(\hat{P}^*_{F,t} - \hat{P}^*_t)] + (1-\mu)[\hat{C}_t - \theta(\hat{P}^*_{F,t} + \hat{S}_t - \hat{P}_t)] \tag{190}$$

$$\Leftrightarrow \hat{Y}^*_t = \mu\hat{C}^*_t + (1-\mu)\hat{C}_t - \theta\mu(\hat{P}^*_{F,t} - \hat{P}^*_t) - \theta(1-\mu)(\hat{P}^*_{F,t} + \hat{S}_t - \hat{P}_t) \tag{191}$$

Reduced System of Equations

We can reduce the system of equations as follows (having 29 equations and variables). The 29 endogenous variables are $P_H, \tilde{P}_H, P, M, Y, A, Z_b, Z_e, \mathit{\Pi}, C$, their foreign counterparts, and $\tau, S, NFA, \alpha_{e_H}, \alpha_{e_F}, \alpha_{b_H}, r_{b_H}, r_{b_F}, r_{e_H}$ and r_{e_F}. The non-portfolio parts of the model consist of the following 17 equations:

$$\hat{P}_t = \hat{P}_{H,t} + (1-\mu)\hat{\tau}_t \tag{192}$$

$$\hat{P}^*_t = \hat{P}^*_{F,t} - (1-\mu)\hat{\tau}_t \tag{193}$$

$$N\hat{F}A_t = \frac{1}{\beta}N\hat{F}A_{t-1} + \hat{Y}_t - \hat{C}_t + \hat{P}_{H,t} - \hat{P}_t + \tilde{\boldsymbol{\alpha}}\hat{\mathbf{r}}_{x,t} \tag{194}$$

$$\rho(E_t\hat{C}_{t+1} - \hat{C}_t) = \rho(E_t\hat{C}^*_{t+1} - \hat{C}^*_t) + (2\mu-1)(E_t\hat{\tau}_{t+1} - \hat{\tau}_t) \tag{195}$$

$$\pi_{H,t} = \frac{1}{\lambda}(\rho\hat{C}_t + \chi\hat{Y}_t + (1-\mu)\hat{\tau}_t - (1+\chi)\hat{A}_t) + \beta E_t\pi_{H,t+1} \tag{196}$$

$$\pi^*_{F,t} = \frac{1}{\lambda}(\rho\hat{C}^*_t + \chi\hat{Y}^*_t - (1-\mu)\hat{\tau}_t - (1+\chi)\hat{A}^*_t) + \beta E_t\pi^*_{F,t+1} \tag{197}$$

$$\hat{M}_t - \hat{P}_t = \hat{C}_t \tag{198}$$

$$\hat{M}^*_t - \hat{P}^*_t = \hat{C}^*_t \tag{199}$$

$$\hat{\tau}_t = \hat{P}^*_{F,t} + \hat{S}_t - \hat{P}_{H,t} \tag{200}$$

$$\pi_{H,t} = \hat{P}_{H,t} - \hat{P}_{H,t-1} \tag{201}$$

$$\pi^*_{F,t} = \hat{P}^*_{F,t} - \hat{P}^*_{F,t-1} \tag{202}$$

$$\hat{Y}_t = \mu\hat{C}_t + (1-\mu)\hat{C}^*_t - \theta\mu(\hat{P}_{H,t} - \hat{P}_t) - \theta(1-\mu)(\hat{P}_{H,t} - \hat{S}_t - \hat{P}^*_t) \tag{203}$$

$$\hat{Y}^*_t = \mu\hat{C}^*_t + (1-\mu)\hat{C}_t - \mu\theta(\hat{P}^*_{F,t} - \hat{P}^*_t) - (1-\mu)\theta(\hat{P}^*_{F,t} + \hat{S}_t - \hat{P}_t) \tag{204}$$

$$\hat{A}_t = \psi_a\hat{A}_{t-1} + u_t \tag{205}$$

$$\hat{A}^*_t = \psi_a\hat{A}^*_{t-1} + u^*_t \tag{206}$$

$$\hat{M}_t = \psi_m\hat{M}_{t-1} + m_t \tag{207}$$

$$\hat{M}^*_t = \psi_m\hat{M}^*_{t-1} - \epsilon\hat{S}_t + m^*_t \tag{208}$$

The portfolio parts of the model consist of the following 9 equations:

$$E_t\hat{r}_{e_H,t+1} = E_t\hat{r}_{b_F,t+1} \tag{209}$$

$$E_t\hat{r}_{e_F,t+1} = E_t\hat{r}_{b_F,t+1} \tag{210}$$

$$E_t\hat{r}_{b_H,t+1} = E_t\hat{r}_{b_F,t+1} \tag{211}$$

$$\hat{r}_{e_H,t+1} = (1-\beta)\hat{\Pi}_{t+1} + \beta\hat{Z}_{e,t+1} - \hat{Z}_{e,t} \tag{212}$$

$$\hat{r}_{e_F,t+1} = (1-\beta)\hat{\Pi}^*_{t+1} + \beta\hat{Z}^*_{e,t+1} - \hat{Z}^*_{e,t} + (2\mu-1)(\hat{\tau}_{t+1} - \hat{\tau}_t) \tag{213}$$

$$\hat{r}_{b_H,t+1} = -\hat{P}_{t+1} - \hat{Z}_{b,t} \tag{214}$$

$$\hat{r}_{b_F,t+1} = -\hat{P}^*_{t+1} - \hat{Z}^*_{b,t} + (2\mu-1)(\hat{\tau}_{t+1} - \hat{\tau}_t) \tag{215}$$

$$\hat{\Pi}_t = \frac{\mathcal{M}}{\mathcal{M}-1}(\hat{P}_{H,t} + \hat{Y}_t - \hat{P}_t) - \frac{1}{\mathcal{M}-1}(\rho\hat{C}_t + (1+\chi)(\hat{Y}_t - \hat{A}_t)) \tag{216}$$

$$\hat{\Pi}^*_t = \frac{\mathcal{M}}{\mathcal{M}-1}(\hat{P}^*_{F,t} + \hat{Y}^*_t - \hat{P}^*_t) - \frac{1}{\mathcal{M}-1}(\rho\hat{C}^*_t + (1+\chi)(\hat{Y}^*_t - \hat{A}^*_t)) \tag{217}$$

E Second Order Approximations of the Portfolio Choice Equations

This section provides a derivation of the second order approximations of the portfolio choice equations. First, I show how to derive the second order approximation of equation (20), which is the optimality condition for the home household's choice of real external holdings of home equity:

$$E_t[C_{t+1}^{-\rho} r_{e_H,t+1}] = E_t[C_{t+1}^{-\rho} r_{b_F,t+1}] \tag{218}$$

We can rewrite this equation as follows:

$$E_t[\exp(\log(C_{t+1}))^{-\rho} \exp(\log(r_{e_H,t+1}))] = E_t[\exp(\log(C_{t+1}))^{-\rho} \exp(\log(r_{b_F,t+1}))] \tag{219}$$

The second order Taylor-series approximation of the above expression is

$$\begin{aligned}
&C^{-\rho} r_{e_H} + E_t[-\rho C^{-\rho} r_{e_H} \hat{C}_{t+1} + C^{-\rho} r_{e_H} \hat{r}_{e_H,t+1}] \\
&+ \frac{1}{2} E_t[\rho^2 C^{-\rho} r_{e_H} \hat{C}_{t+1}^2 - 2\rho C^{-\rho} r_{e_H} \hat{C}_{t+1} \hat{r}_{e_H,t+1} + C^{-\rho} r_{e_H} \hat{r}_{e_H,t+1}^2] \\
&= C^{-\rho} r_{b_F} + E_t[-\rho C^{-\rho} r_{b_F} \hat{C}_{t+1} + C^{-\rho} r_{b_F} \hat{r}_{b_F,t+1}] \\
&+ \frac{1}{2} E_t[\rho^2 C^{-\rho} r_{b_F} \hat{C}_{t+1}^2 - 2\rho C^{-\rho} r_{b_F} \hat{C}_{t+1} \hat{r}_{b_F,t+1} + C^{-\rho} r_{b_F} \hat{r}_{b_F,t+1}^2]
\end{aligned} \tag{220}$$

Note that $\hat{C}_{t+1} = \log C_{t+1} - \log C$, the derivative is taken w.r.t. $\log C_{t+1}$, evaluated at $\log C$. We can rearrange and find

$$\begin{aligned}
&1 + E_t[-\rho \hat{C}_{t+1} + \hat{r}_{e_H,t+1}] + \frac{1}{2} E_t[\rho^2 \hat{C}_{t+1}^2 - 2\rho \hat{C}_{t+1} \hat{r}_{e_H,t+1} + \hat{r}_{e_H,t+1}^2] \\
&= 1 + E_t[-\rho \hat{C}_{t+1} + \hat{r}_{b_F,t+1}] + \frac{1}{2} E_t[\rho^2 \hat{C}_{t+1}^2 - 2\rho \hat{C}_{t+1} \hat{r}_{b_F,t+1} + \hat{r}_{b_F,t+1}^2]
\end{aligned} \tag{221}$$

$$\Leftrightarrow E_t[\hat{r}_{xe_H,t+1} - \rho \hat{C}_{t+1} \hat{r}_{xe_H,t+1} + \frac{1}{2}[\hat{r}_{e_H,t+1}^2 - \hat{r}_{b_F,t+1}^2]] = 0 \tag{222}$$

Remember that $r_{b_F}, = r_{e_H}$, $\hat{r}_{xe_H,t+1} = \hat{r}_{e_H,t+1} - \hat{r}_{b_F,t+1}$. The last equation is identical to equation (46). In a similar way we can derive equations (47) and (48).

Second, I show how to derive the second order approximation of equation (104) which is the optimality condition for the foreign household's choice of real external holdings of home equity:

$$E_t[C_{t+1}^{*-\rho} r_{e_H,t+1} \frac{1}{Q_{t+1}}] = E_t[C_{t+1}^{*-\rho} r_{b_F,t+1} \frac{1}{Q_{t+1}}] \tag{223}$$

$$\begin{aligned}
\Leftrightarrow\ & E_t[\exp(\log(C_{t+1}^*))^{-\rho} \exp(\log(r_{e_H,t+1})) \frac{1}{\exp(\log(Q_{t+1}))}] = \\
& E_t[\exp(\log(C_{t+1}^*))^{-\rho} \exp(\log(r_{b_F,t+1})) \frac{1}{\exp(\log(Q_{t+1}))}]
\end{aligned} \tag{224}$$

The second order approximation of the above expression is

$$
\begin{aligned}
&C^{*-\rho}r_{e_H}\frac{1}{Q}+E_t[-\rho C^{*-\rho}r_{e_H}\frac{1}{Q}\hat{C}^*_{t+1}+C^{*-\rho}r_{e_H}\frac{1}{Q}\hat{r}_{e_H,t+1}-C^{*-\rho}r_{e_H}\frac{1}{Q}\hat{Q}_{t+1}]\\
&+\frac{1}{2}E_t[\rho^2C^{*-\rho}r_{e_H}\frac{1}{Q}\hat{C}^{*2}_{t+1}-2\rho C^{*-\rho}r_{e_H}\frac{1}{Q}\hat{C}^*_{t+1}\hat{r}_{e_H,t+1}+C^{*-\rho}r_{e_H}\frac{1}{Q}\hat{r}^2_{e_H,t+1}]\\
&+\frac{1}{2}E_t[2\rho C^{*-\rho}r_{e_H}\frac{1}{Q}\hat{C}^*_{t+1}\hat{Q}_{t+1}-2C^{*-\rho}r_{e_H}\frac{1}{Q}\hat{Q}_{t+1}\hat{r}_{e_H,t+1}+C^{*-\rho}r_{e_H}\frac{1}{Q}\hat{Q}^2_{t+1}]\\
&=C^{*-\rho}r_{b_F}\frac{1}{Q}+E_t[-\rho C^{*-\rho}r_{b_F}\frac{1}{Q}\hat{C}^*_{t+1}+C^{*-\rho}r_{b_F}\frac{1}{Q}\hat{r}_{b_F,t+1}-C^{*-\rho}r_{b_F}\frac{1}{Q}\hat{Q}_{t+1}]\\
&+\frac{1}{2}E_t[\rho^2C^{*-\rho}r_{b_F}\frac{1}{Q}\hat{C}^{*2}_{t+1}-2\rho C^{*-\rho}r_{b_F}\frac{1}{Q}\hat{C}^*_{t+1}\hat{r}_{b_F,t+1}+C^{*-\rho}r_{b_F}\frac{1}{Q}\hat{r}^2_{b_F,t+1}]\\
&+\frac{1}{2}E_t[2\rho C^{*-\rho}r_{b_F}\frac{1}{Q}\hat{C}^*_{t+1}\hat{Q}_{t+1}-2C^{*-\rho}r_{b_F}\frac{1}{Q}\hat{Q}_{t+1}\hat{r}_{b_F,t+1}+C^{*-\rho}r_{b_F}\frac{1}{Q}\hat{Q}^2_{t+1}] \qquad (225)
\end{aligned}
$$

We can rearrange and find

$$
\begin{aligned}
&1+E_t[-\rho\hat{C}^*_{t+1}+\hat{r}_{e_H,t+1}-\hat{Q}_{t+1}]+\frac{1}{2}E_t[\rho^2\hat{C}^{*2}_{t+1}-2\rho\hat{C}^*_{t+1}\hat{r}_{e_H,t+1}+\hat{r}^2_{e_H,t+1}]\\
&+\frac{1}{2}E_t[2\rho\hat{C}^*_{t+1}\hat{Q}_{t+1}-2\hat{Q}_{t+1}\hat{r}_{e_H,t+1}+\hat{Q}^2_{t+1}]\\
&=1+E_t[-\rho\hat{C}^*_{t+1}+\hat{r}_{b_F,t+1}-\hat{Q}_{t+1}]+\frac{1}{2}E_t[\rho^2\hat{C}^{*2}_{t+1}-2\rho\hat{C}^*_{t+1}\hat{r}_{b_F,t+1}+\hat{r}^2_{b_F,t+1}]\\
&+\frac{1}{2}E_t[2\rho\hat{C}^*_{t+1}\hat{Q}_{t+1}-2\hat{Q}_{t+1}\hat{r}_{b_F,t+1}+\hat{Q}^2_{t+1}] \qquad (226)\\
\Leftrightarrow &E_t[\hat{r}_{xe_H,t+1}-\rho\hat{C}^*_{t+1}\hat{r}_{xe_H,t+1}+\frac{1}{2}[\hat{r}^2_{e_H,t+1}-\hat{r}^2_{b_F,t+1}]-\hat{Q}_{t+1}\hat{r}_{xe_H,t+1}]=0 \qquad (227)
\end{aligned}
$$

The last equation is identical to equation (49). In a similar way we can derive equations (50) and (51).

F Asset Prices and Excess Returns

The log changes in real returns on home and foreign equities and bonds are given by equations (140)-(143). Taking the differences between these changes and the return on the numéraire asset (the foreign bond) yields

$$\hat{r}_{xe_H,t+1}=(1-\beta)\hat{\Pi}_{t+1}+\beta\hat{Z}_{e,t+1}-\hat{Z}_{e,t}+\hat{P}^*_{t+1}+\hat{Z}^*_{b,t}-(\hat{Q}_{t+1}-\hat{Q}_t) \qquad (228)$$

$$\hat{r}_{xe_F,t+1}=(1-\beta)\hat{\Pi}^*_{t+1}+\beta\hat{Z}^*_{e,t+1}-\hat{Z}^*_{e,t}+\hat{P}^*_{t+1}+\hat{Z}^*_{b,t} \qquad (229)$$

$$\hat{r}_{xb_H,t+1}=-\hat{P}_{t+1}-\hat{Z}_{b,t}+\hat{P}^*_{t+1}+\hat{Z}^*_{b,t}-(\hat{Q}_{t+1}-\hat{Q}_t) \qquad (230)$$

Up to a first order approximation $\hat{r}_{xk,t} \quad \forall k\in K'$ is a mean zero i.i.d. variable, i.e.,

$$E_t\hat{r}_{xe_H,t+1}=(1-\beta)E_t\hat{\Pi}_{t+1}+E_t\beta\hat{Z}_{e,t+1}-\hat{Z}_{e,t}+E_t\hat{P}^*_{t+1}+\hat{Z}^*_{b,t}-E_t(\hat{Q}_{t+1}-\hat{Q}_t)=0 \qquad (231)$$

$$E_t\hat{r}_{xe_F,t+1}=(1-\beta)E_t\hat{\Pi}^*_{t+1}+E_t\beta\hat{Z}^*_{e,t+1}-\hat{Z}^*_{e,t}+E_t\hat{P}^*_{t+1}+\hat{Z}^*_{b,t}=0 \qquad (232)$$

$$E_t\hat{r}_{xb_H,t+1}=-E_t\hat{P}_{t+1}-\hat{Z}_{b,t}+E_t\hat{P}^*_{t+1}+\hat{Z}^*_{b,t}-E_t(\hat{Q}_{t+1}-\hat{Q}_t)=0 \qquad (233)$$

Rewriting these equations yields

$$\hat{Z}_{e,t} - \hat{Z}^*_{b,t} - \hat{Q}_t = (1-\beta)E_t\hat{\Pi}_{t+1} + E_t\beta\hat{Z}_{e,t+1} + E_t\hat{P}^*_{t+1} - E_t\hat{Q}_{t+1} \tag{234}$$

$$\hat{Z}^*_{e,t} - \hat{Z}^*_{b,t} = (1-\beta)E_t\hat{\Pi}^*_{t+1} + E_t\beta\hat{Z}^*_{e,t+1} + E_t\hat{P}^*_{t+1} \tag{235}$$

$$\hat{Z}_{b,t} - \hat{Z}^*_{b,t} - \hat{Q}_t = -E_t\hat{P}_{t+1} + E_t\hat{P}^*_{t+1} - E_t\hat{Q}_{t+1} \tag{236}$$

Plugging the period t values on the right-hand side of the above equations in equations (228)-(230) yields

$$\begin{aligned}
\hat{r}_{xe_H,t+1} &= (1-\beta)\hat{\Pi}_{t+1} + \beta\hat{Z}_{e,t+1} - E_t((1-\beta)\hat{\Pi}_{t+1} - \beta\hat{Z}_{e,t+1}) \\
&\quad + (\hat{P}^*_{t+1} - \hat{P}^*_t) - E_t(\hat{P}^*_{t+1} - \hat{P}^*_t) - (\hat{Q}_{t+1} - E_t\hat{Q}_{t+1}) \\
\hat{r}_{xe_F,t+1} &= (1-\beta)\hat{\Pi}^*_{t+1} + \beta\hat{Z}^*_{e,t+1} - ((1-\beta)E_t\hat{\Pi}^*_{t+1} + E_t\beta\hat{Z}^*_{e,t+1}) && (237) \\
&\quad + (\hat{P}^*_{t+1} - \hat{P}^*_t) - E_t(\hat{P}^*_{t+1} - \hat{P}^*_t) && (238) \\
\hat{r}_{xb_H,t+1} &= -\hat{P}_{t+1} + \hat{P}^*_{t+1} - \hat{Q}_{t+1} + E_t\hat{P}_{t+1} - E_t\hat{P}^*_{t+1} + E_t\hat{Q}_{t+1} && (239)
\end{aligned}$$

Since $\hat{Q}_t = \hat{S}_t + \hat{P}^*_t - \hat{P}_t$, the last equation simplifies to

$$\hat{r}_{xb_H,t+1} = -(\hat{S}_{t+1} - E_t\hat{S}_{t+1}) \tag{240}$$

The difference in changes in the home and foreign bond return results from unanticipated changes in the nominal exchange rate.
From the pricing of equity we know that (see equations (140)-(143))

$$\hat{Z}_{e,t} = (1-\beta)\hat{\Pi}_{t+1} + \beta\hat{Z}_{e,t+1} - \hat{r}_{e_H,t+1} \tag{241}$$

$$\hat{Z}^*_{e,t} = (1-\beta)\hat{\Pi}^*_{t+1} + \beta\hat{Z}^*_{e,t+1} - \hat{r}_{e_F,t+1} + \hat{Q}_{t+1} - \hat{Q}_t \tag{242}$$

$$\hat{Z}_{b,t} = -\hat{P}_{t+1} - \hat{r}_{b_H,t+1} \tag{243}$$

$$\hat{Z}^*_{b,t} = -\hat{P}^*_{t+1} - \hat{r}_{b_F,t+1} + \hat{Q}_{t+1} - \hat{Q}_t \tag{244}$$

Taking expectations yields

$$\hat{Z}_{e,t} = (1-\beta)E_t\hat{\Pi}_{t+1} + \beta E_t\hat{Z}_{e,t+1} - E_t\hat{r}_{e_H,t+1} \tag{245}$$

$$\hat{Z}^*_{e,t} = (1-\beta)E_t\hat{\Pi}^*_{t+1} + \beta E_t\hat{Z}^*_{e,t+1} - E_t\hat{r}_{e_F,t+1} + E_t\hat{Q}_{t+1} - \hat{Q}_t \tag{246}$$

$$\hat{Z}_{b,t} = -E_t\hat{P}_{t+1} - E_t\hat{r}_{b_H,t+1} \tag{247}$$

$$\hat{Z}^*_{b,t} = -E_t\hat{P}^*_{t+1} - E_t\hat{r}_{b_F,t+1} + E_t\hat{Q}_{t+1} - \hat{Q}_t \tag{248}$$

Using equations (146) and (150) we can rewrite these equations as follows

$$\hat{Z}_{e,t} = (1-\beta)E_t\hat{\Pi}_{t+1} + \beta E_t\hat{Z}_{e,t+1} - \rho(E_t\hat{C}_{t+1} - \hat{C}_t) \tag{249}$$

$$\hat{Z}^*_{e,t} = (1-\beta)E_t\hat{\Pi}^*_{t+1} + \beta E_t\hat{Z}^*_{e,t+1} - \rho(E_t\hat{C}^*_{t+1} - \hat{C}^*_t) \tag{250}$$

$$\hat{Z}_{b,t} = -E_t\hat{P}_{t+1} - \rho(E_t\hat{C}_{t+1} - \hat{C}_t) \tag{251}$$

$$\hat{Z}^*_{b,t} = -E_t\hat{P}^*_{t+1} - \rho(E_t\hat{C}^*_{t+1} - \hat{C}^*_t) \tag{252}$$

These equations are identical to equations (84)-(87).

G Impulse Response Functions

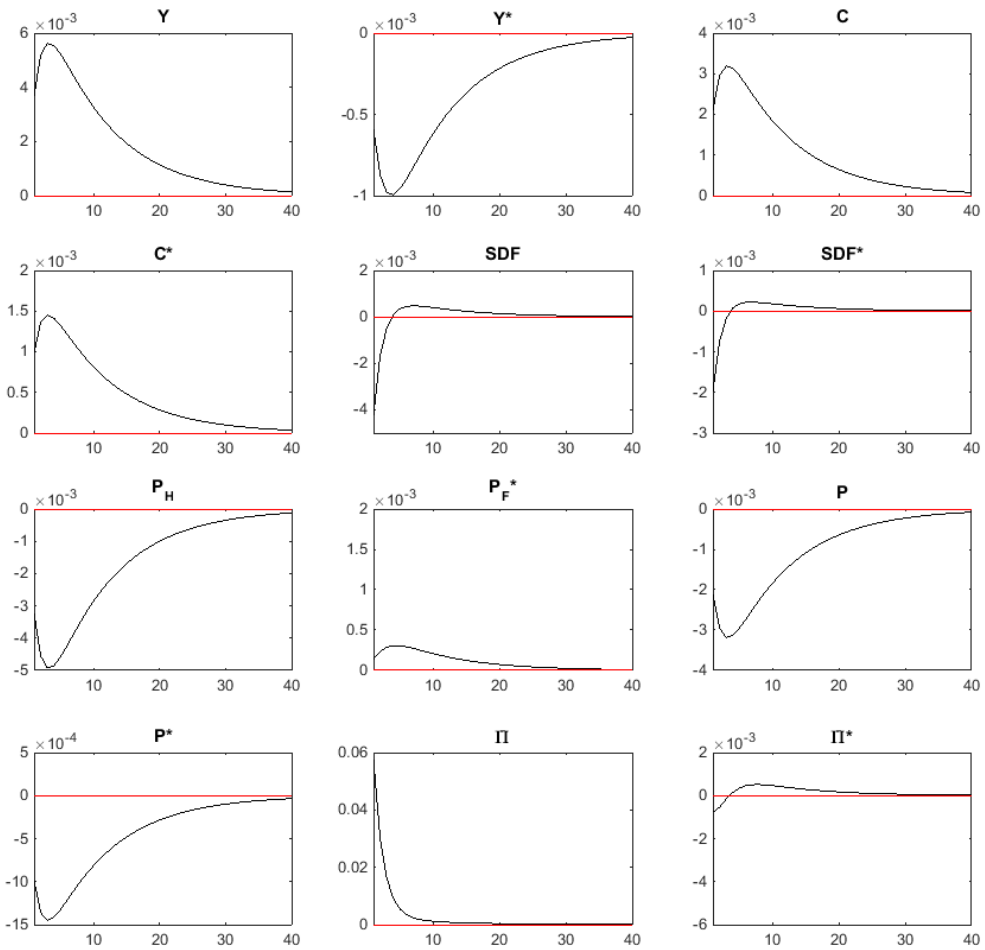

Figure A.1: Impulse responses to a home productivity shock of one standard deviation under a flexible exchange rate regime. The values represent the percentage deviation from the steady state (exceptions are SDF and SDF^*, which are defined as $-\rho(\hat{C}_{t+1} - \hat{C}_t)$ and $-\rho(\hat{C}^*_{t+1} - \hat{C}^*_t)$, respectively). For each variable the same scale for the y-axes as in Figure A.3 (fixed exchange rate regime) is used to allow for comparisons of impulse responses across regimes.

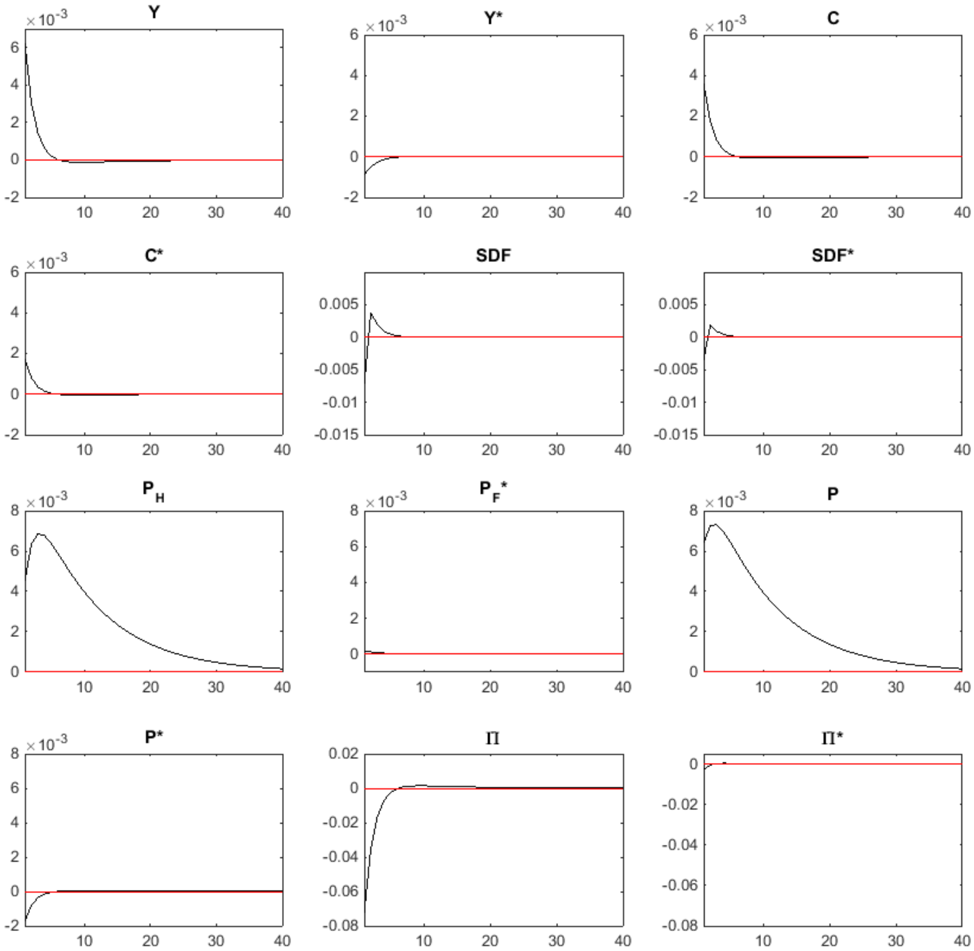

Figure A.2: Impulse responses to a home monetary shock of one standard deviation under a flexible exchange rate regime. The values represent the percentage deviation from the steady state (exceptions are SDF and SDF^*, which are defined as $-\rho(\hat{C}_{t+1} - \hat{C}_t)$ and $-\rho(\hat{C}^*_{t+1} - \hat{C}^*_t)$, respectively). For each variable the same scale for the y-axes as in Figure A.4 (fixed exchange rate regime) is used to allow for comparisons of impulse responses across regimes.

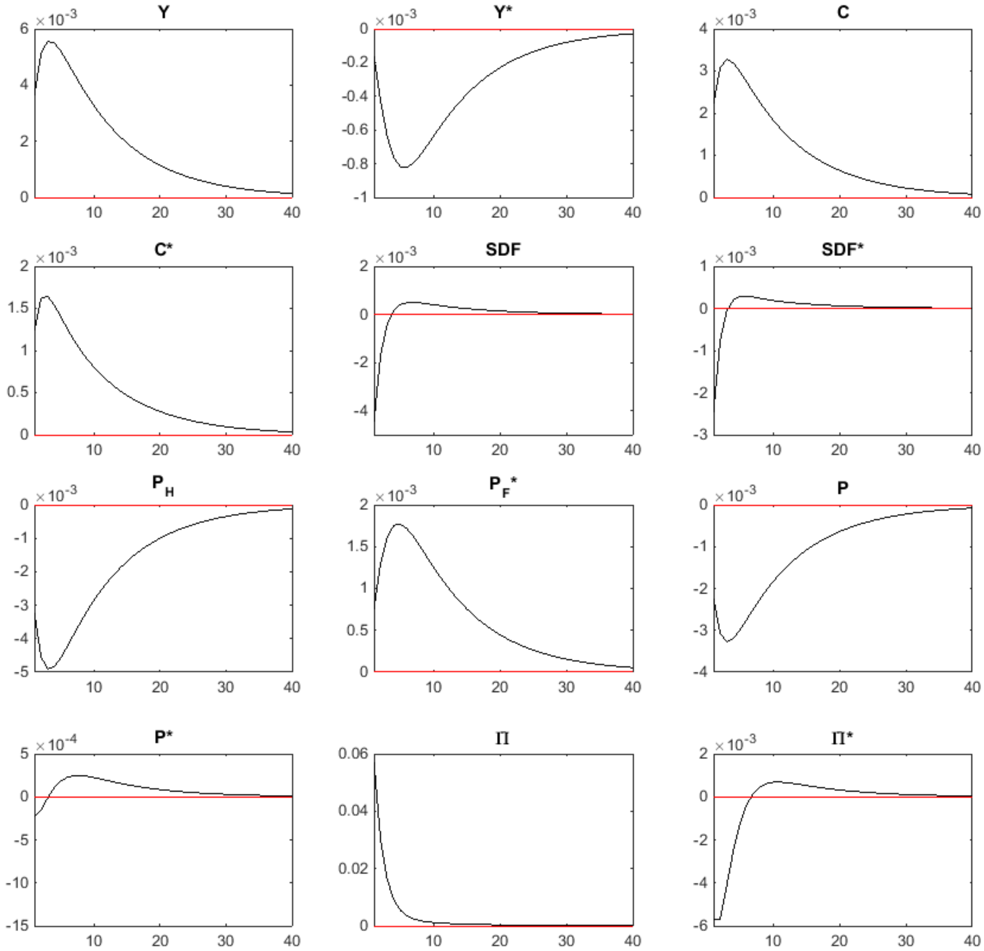

Figure A.3: Impulse responses to a home productivity shock of one standard deviation under a fixed exchange rate regime. The values represent the percentage deviation from the steady state (exceptions are SDF and SDF^*, which are defined as $-\rho(\hat{C}_{t+1} - \hat{C}_t)$ and $-\rho(\hat{C}^*_{t+1} - \hat{C}^*_t)$, respectively). For each variable the same scale for the y-axes as in Figure A.1 (flexible exchange rate regime) is used to allow for comparisons of impulse responses across regimes.

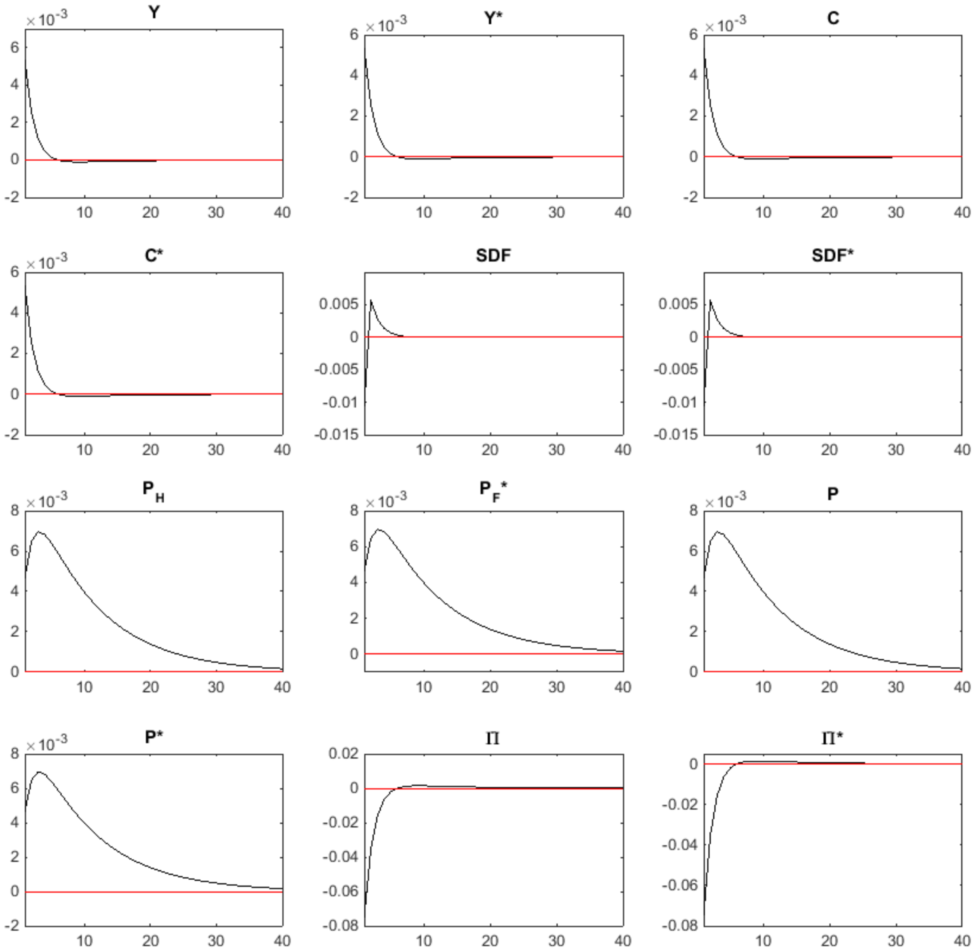

Figure A.4: Impulse responses to a home monetary shock of one standard deviation under a fixed exchange rate regime. The values represent the percentage deviation from the steady state (exceptions are SDF and SDF^*, which are defined as $-\rho(\hat{C}_{t+1} - \hat{C}_t)$ and $-\rho(\hat{C}^*_{t+1} - \hat{C}^*_t)$, respectively). For each variable the same scale for the y-axes as in Figure A.2 (flexible exchange rate regime) is used to allow for comparisons of impulse responses across regimes.