

# Unconventional Monetary Policy under Appreciation Pressure – The Role of Financial Frictions

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# Unconventional Monetary Policy under Appreciation Pressure The Role of Financial Frictions \*

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#### Abstract

We build a two-country model with imperfect financial intermediation. Banks face limits to arbitrage which lead to positive excess returns in the investment markets and a risk premium in the international credit market. Gross capital flows affect the exchange rate since banks are balance sheet constrained and can only absorb additional flows on the international credit market if the exchange rate adjusts. Similarly, unconventional monetary policies such as foreign exchange interventions and credit easing influence asset prices in financial markets where banks are credit constrained. Within this framework, we study three external sources of appreciation pressure: Financial frictions in the foreign investment market, financial frictions in the international credit market and capital inflow shocks. In the two latter cases, foreign exchange interventions can reverse the resulting exchange rate movements and misallocations of capital. Furthermore, under certain conditions, foreign exchange interventions and credit easing are substitutes since asset purchases in one market reduce the excess returns in both.

**JEL-Classification:** E44 E52 F31 F32 F41 G11 G15 G20

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#### 1 Introduction

The recent global financial crisis and the economic downturn that came along with it made many of the world's leading central banks lower their policy rate to very low levels with the aim to stabilize the financial system, mitigate the economic downturn and counter deflationary risks. However, no satisfactory recovery was accomplished. Liquidity provided by the central banks did not reach the private sector as there was a significant disruption in financial intermediation leading to a credit market freeze, which was reflected in various wide credit spreads. This turmoil on global financial markets brought along additional challenges for countries like Switzerland, Denmark or Israel that had to face a further tightening of the monetary conditions due to the intense appreciation pressure on their currencies. Given the limited effectiveness of low short-term interest rates during this crisis, and given that the central bank's policy rates soon reached the effective lower bound where the options available under conventional monetary policy are exhausted, many central banks started to use unconventional measures.

Various forms of unconventional monetary policy have been put into practice, among them quantitative easing (QE) and credit easing (CE) as introduced by the Federal Reserve, the Bank of England, the Bank of Japan and the ECB, and foreign exchange interventions (FXI), as used by the Swiss National Bank or the Bank of Japan<sup>2</sup>. Bernanke and Reinhart (2004) define QE as a policy where the central bank shifts its focus from the price of reserves (the short-term policy rate) to the quantity of reserves to affect the monetary base. In particular, the central bank increases the size of its balance sheet beyond the level needed to set the policy rate at zero. This is usually done by the acquisition of domestic medium-term and longer-term securities. Credit easing can be described as a special case of QE if it also involves an increase in the monetary base. However, the intentions of credit easing programs differ from the ones of QE programs. Their goal is to change the asset composition on the central bank's balance sheet in order to reduce specific interest rates or restore the functioning of specific markets (see Bernanke (2009)). When doing FXI, the central bank purchases foreign exchange with the goal of depreciating the domestic currency, which in turn is meant to relax the monetary conditions and stimulate the economy.

In this paper, we develop a theoretical framework with financial frictions that allows to compare credit easing policies to foreign exchange interventions. The model provides intuition for how central bank intermediation can help to reduce these financial frictions, which are reflected in positive credit spreads and spreads in the interest parity, and hence mitigate their negative consequences on investment and international trade. Our framework reveals that appreciation pressure can be caused by financial frictions on the international credit market and the foreign investment market as well as large capital inflows and allows to illustrate how the central bank can respond to this pressure using the two unconventional tools under analysis.

There already exists substantial research on unconventional monetary policy tools. While most

<sup>&</sup>lt;sup>1</sup>Under conventional monetary policy the economy is managed via the short-term interest rates.

<sup>&</sup>lt;sup>2</sup>The Swedish Riksbank has recently announced to be willing to intervene on the foreign exchange market if necessary, to safeguard the rise in inflation.

of the literature on QE or asset purchase programs emerged after the financial crisis of 2007-2009<sup>3</sup>, the literature on FXI goes further back as this instrument was already applied after the breakdown of the Bretton Woods system. In both literature streams, we find models relying on financial frictions modelled as limited commitment of financial intermediaries. In the past few years, financial frictions and imperfect financial intermediation have got a lot of attention and have become an important modelling tool as frictions in financial markets are considered to be one of the main reasons for the strong spreading of the recent financial crisis. On the one hand, they provide a plausible explanation for spreads in credit rates. But on the other hand, they also generate a portfolio balance channel through which QE or FXI can be effective. In a few words, the portfolio balance channel can be described as follows: When purchasing a particular type of asset, the central bank reduces the supply of this asset left for absorption by the private sector. In order for investors to be willing to reduce its amount of this asset in their portfolio, the price of the asset has to increase and its expected return to fall. This effect will spill over to other types of assets to the extent that investors rebalance their portfolio by replacing the asset purchased by the central bank by other types of assets. Thereby, prices of the latter will rise and their returns decline as well. Imperfect substitutability between the asset the central bank purchases and the one it uses to finance its expenses (usually reserves) is the key feature for the portfolio balance channel to work. In QE models, different types of domestic assets are modelled as imperfect substitutes, while in FXI models domestic and foreign assets are imperfectly substitutable. Despite these similarities in the literature of QE and FXI, what is missing to our knowledge is a framework that allows to directly compare the usage of these two tools. Our paper represents a first contribution to fill this gap.

The empirical literature on QE or large scale asset purchase programs as a monetary policy tool assesses the effectiveness of such programs and the channels through which QE may work.<sup>4</sup> Overall, there is broad evidence that QE programs were successful at flattening the yield curve, while the precise mechanism through which it works remains unclear. Theory suggests two main channels through which QE may affect interest rates, the signalling channel (the intervention is a signal about the future stance of monetary policy, see, e.g., Eggertsson and Woodford (2003)), or the portfolio balance channel.<sup>5</sup> The portfolio balance channel was first described by Tobin (1958, 1969). Later, various theoretical foundations for imperfect asset substitutability have emerged. Andrés et al. (2004) introduce an adjustment to household preferences to allow for imperfect asset substitutability between holdings of long-term bonds and money in a New Keynesian model. Similarly, using a partial equilibrium approach, Vayanos and Vila (2009) propose a model where investors have heterogeneous preferences for assets of different maturities ("preferred habitat" motive). Gertler and Karadi (2013) follow Gertler and Karadi (2011) and Gertler and Kiyotaki (2011)

<sup>&</sup>lt;sup>3</sup>The first studies on QE analyze Japan's experiences after it hit the zero-lower bound in 1999.

<sup>&</sup>lt;sup>4</sup>See, e.g., Baumeister and Benati (2013) for the US and the UK, Gambacorta et al. (2014) for several countries, D'Amico and King (2013), Doh (2010), Gagnon et al. (2011a), Hamilton and Wu (2012), Krishnamurthy and Vissing-Jorgensen (2011) for the US, and Meier (2009) and Joyce et al. (2011) for the UK.

<sup>&</sup>lt;sup>5</sup>Although under QE the central bank extends the quantity of reserves only few studies discuss the liquidity channel through which QE may work (see, e.g., Christensen and Krogstrup (2016)).

by looking at QE as a form of financial intermediation, performed by the central bank. The central bank acquires assets by issuing interest-bearing short-term debt. Limits to arbitrage in private financial intermediation caused by financial frictions result in imperfect substitutability between the assets the government purchases and reserves. Such limits to arbitrage on the one hand lead to extraordinary returns on assets and on the other hand generate a role for central bank intermediation to drive these returns down. As the central bank can intermediate long-term government bonds and securities with some private risks, the model presents a unified approach to analyze a variety of programs used in practice. While most of the theoretical literature models asset purchase programs in closed economies, Dedola et al. (2013) provide a two-country model that allows to analyze the international dimension of unconventional polices in economies with financial frictions, but do not study exchange rates.

Many empirical studies analyze the effectiveness of FXI at influencing the exchange rate. There is a consensus that non-sterilized interventions do have an impact as they change the monetary base. The effectiveness of sterilized interventions is less clear on both theoretical and empirical grounds. FXI may affect the exchange rate, the signalling channel and the portfolio balance channel. Early theoretical foundations for the portfolio balance channel were provided by Henderson and Rogoff (1981), Kouri (1976) and Branson and Henderson (1985). More recent advances are Kumhof (2010) and Gabaix and Maggiori (2015). Gabaix and Maggiori (2015)'s model of exchange rate determination is a modern version of the traditional portfolio balance models. It illustrates how gross capital flows matter for the determination of exchange rates and allows to investigate the effects of foreign exchange interventions. Similar to Gertler and Karadi (2013), they use limited commitment of financial intermediaries to introduce financial frictions and to endogenize a spread in the interest rate parity, reflecting a currency risk premium.

This paper combines the idea of Gertler and Karadi (2013) and Gabaix and Maggiori (2015). We use a simplified, real version of the model by Gertler and Karadi (2013) and extend it to an open economy model. As done in the two papers, we introduce an agency problem between borrowers and lenders which generates a portfolio balance channel. Limited commitment of private banks leads to an endogenous credit constraint and results in limits to arbitrage in the markets for investment and in the international credit market reflected in excess returns (over the riskless rate) on these assets. More precisely, compared to a frictionless equilibrium, capital costs in the investment markets are higher and in the international credit market there is a deviation from the interest rate parity (IP). This theoretical framework allows to understand the role of credit easing versus FXI when a portfolio balance channel is at work. Central bank intermediation will only have an impact on prices if banks are balance sheet constrained.

We analyze the usage of the two tools starting from a country that faces an appreciation. We

<sup>&</sup>lt;sup>6</sup>For a literature review see, e.g., Sarno and Taylor (2001), Neely (2005) or Menkhoff (2010).

<sup>&</sup>lt;sup>7</sup>Under non-sterilized FXI the central bank purchases foreign-currency denominated assets which leads to an increase in the monetary base whereas under sterilized FXI in addition to the first transaction the central bank sells a corresponding quantity of domestic-currency assets in order to reverse the effects on the monetary base.

characterize different external sources and types of appreciation pressure and analyze under what conditions which type of unconventional monetary policy can be a response to it. We identify three sources of external appreciation pressure related to financial frictions. They involve either an increase in these frictions or a change in gross capital flows. The first one is financial frictions in the international credit market. This comes along with a decrease in the home banks' willingness to bear exchange rate risk and to absorb the excess supply of foreign bonds resulting from trade and financial imbalances. Less intermediation in the international credit market leads to a deviation from IP and brings along a home real appreciation. Second, we look at capital inflow shocks and show that if banks are credit constrained in the international credit market gross capital inflows result in an increase in the IP spread and therefore an appreciation since such inflows absorb a large part of the home banks' limited intermediation capacity. Finally, there might be financial frictions in the foreign investment market. When foreign banks are less able to intermediate funds, investment in the foreign country decreases, while the home country reduces its net exports by increasing its consumption and its own investment. The higher relative level of future output induces a permanent home appreciation, but does not lead to a deviation from IP.

We show that within our model the home central bank can use unconventional monetary policy to reduce the appreciation pressure in the first two cases only. Since financial frictions in the foreign investment market lead to a permanent home appreciation, unconventional policy, if effective at all, can lower the appreciation today only at the cost of higher future appreciation. Financial frictions in the international credit market and capital inflow shocks, on the other hand, both lead to a temporary appreciation due to an increase in the IP spread, i.e. the excess return on foreign bonds. By acquiring foreign bonds and issuing domestic bonds the central bank increases overall financial intermediation and thereby helps to reduce excess returns and bring the home country's economy closer to the frictionless state. The crucial assumption is that unlike private intermediaries, the central bank is not balance sheet constrained. Finally, we argue that credit easing can achieve the same goal as foreign exchange interventions if banks are also credit constrained on the domestic investment market. Such spill-over effects result from the fact that banks adjust their portfolio in response to central bank intermediation. Interventions in one market make the banks shift their assets to the other market, reducing the excess returns in both markets. The central bank's intervention should target the market that exhibits the highest excess returns as this will make sure that the balance sheet extension of the central bank needed to reach its goal will be minimized.

The structure of the paper is as follows. Section 2 presents our basic model set-up. In section 3, we provide some first intuition by considering the model in a frictionless environment. Section 4 discusses the role of the various types of financial frictions and section 5 shows how capital inflows can be a source of appreciation pressure. In section 6, we introduce central bank intermediation in the form of credit easing and FXI. Section 7 analyzes the home country's possible policy response to different sources of appreciation pressure and section 8 concludes.

#### 2 Model

Our general equilibrium model combines the main elements of Gertler and Karadi (2013) and Gabaix and Maggiori (2015). In both of them, financial frictions are the key model feature and are modelled as an agency problem between financial intermediaries and their creditors, thereby generating a portfolio balance channel. As a starting point, we take a simplified version of the closed-economy model by Gertler and Karadi (2013), breaking it down to 2 periods (t = 0, 1) and assuming a deterministic and real model environment. We extend this set-up to a two-country model with one homogeneous traded and, per country, one non-traded good, using the price of the nontraded goods as the numéraires. In each country, there are households, firms and banks, who intermediate between households and firms as well as between agents of the two countries. At a later point, we will also introduce a central bank.

By choosing a real set-up, we abstract from exchange rate movements stemming from monetary phenomena and nominal frictions and therefore can study the credit channel in isolation from any other influences, which makes it more easily comprehensible. Even tough minimalistic, our model is meant to capture the fundamental structure of the domestic and international bond markets, where financial intermediation plays a key role, and contains the main economic intuition.

#### 2.1 Households

In each of the two countries, there is a continuum of identical households that have unit mass. The representative household in the home country works, consumes and saves. He is endowed with nontraded goods in both and traded goods in the first period(s). He provides labor in two ways, on the one hand the household runs a bank and on the other hand, he works for the non-financial firm (in the second period only).<sup>8</sup> The supply of labor to the non-financial firm L is inelastic. The household saves in the first period by on the one hand transferring some exogenous amount  $N_0$  of his tradable goods endowment as seed capital to its bank and on the other hand buying domestic bonds issued by a bank other than the one he owns. The household consumes the consumption basket  $C_t$ , which is a composite of nontradable goods consumption  $C_{NT,t}$  and tradable goods consumption  $C_{T,t}$ . The consumption index is of Cobb-Douglas form<sup>9</sup>

$$C_t = \left(C_{NT,t}^{\chi} C_{T,t}\right)^{\frac{1}{1+\chi}} \tag{1}$$

<sup>&</sup>lt;sup>8</sup>To make this set-up more realistic, we could also model the households to consist of workers and bankers as Gertler and Karadi (2013) do.

<sup>&</sup>lt;sup>9</sup>The Cobb-Douglas utility function implies that income elasticities for both goods are unitary and the budget shares are independent of the income level of the consumer.

We consider a log utility function. The households' optimization problem is

$$\max_{B_0,(C_{NT,t},C_{T,t})_{t=0,1}} \ln C_0 + \beta \ln C_1$$
  
subject to (1)  
$$P_0C_0 + B_0 = p_0Y_{T,0} - p_0N_0 + Y_{NT,0}$$
 (2)

$$P_1C_1 = R_1B_0 + w_1L + p_1N_1 + Y_{NT,1}$$
(3)

where  $P_t$  is the price index,  $Y_{NT,t}$  and  $Y_{T,0}$  are the endowments of the nontraded and the traded good, and  $B_0$  are bond holdings. Note that while these bonds represent a claim on traded goods, they are expressed in terms of the domestic non-traded good.  $p_tN_t$  are transfers to and from the household's bank and taken as given by the household.  $p_t$  is the price of the traded good,  $w_1$  is the wage and  $R_1$  is the gross return on bond holdings, all measured in terms of the numéraire, i.e., the domestic non-tradable good. The price index is defined as the minimum cost, in terms of the numéraire, of obtaining one unit of the consumption basket. Thus, given the optimal choice of  $C_{NT,t}$  and  $C_{T,t}$ , total consumption expenditure is

$$P_t C_t = C_{NT,t} + p_t C_{T,t} \tag{4}$$

The maximization problem of the household can be split into an intertemporal and an intratemporal problem. In the former, the household makes his consumption/savings decision independent of the division of consumption expenditure between tradables and nontradables and in the latter he decides on consumption of traded and nontraded goods for a given level of consumption expenditure.

The intertemporal problem is

$$\max_{B_0,C_0,C_1} \ln C_0 + \beta \ln C_1$$
 subject to 
$$P_0C_0 + B_0 = p_0Y_{T,0} - p_0N_0 + Y_{NT,0}$$
 
$$P_1C_1 = R_1B_0 + w_1L + p_1N_1 + Y_{NT,1}$$

The first order conditions are

$$C_t: \qquad \frac{1}{C_t} = P_t \Lambda_t, \qquad t = 0, 1 \tag{5}$$

$$A_0 = R_1 \beta \Lambda_1 \tag{6}$$

where  $\Lambda_t$  is the Lagrange multiplier attached to the period-t budget constraint. Combining the first order conditions yields the Euler condition

$$1 = R_1 \Lambda_{0,1} \tag{7}$$

where  $\Lambda_{0,1} = \beta \frac{\Lambda_1}{\Lambda_0} = \beta \frac{P_0 C_0}{P_1 C_1}$  is the household's intertemporal marginal rate of substitution.

The intratemporal problem is

$$\max_{C_{NT}, C_T} \left( C_{NT}^{\chi} C_T \right)^{\frac{1}{1+\chi}}$$
 subject to (4)

where total consumption expenditure PC is taken as given. The first order conditions are

$$C_{NT}:$$
 
$$\frac{\chi}{1+\chi} \left(\frac{C_T}{C_{NT}}\right)^{\frac{1}{1+\chi}} = \mu \tag{8}$$

$$C_T: \qquad \frac{1}{1+\chi} \left(\frac{C_{NT}}{C_T}\right)^{\frac{\chi}{1+\chi}} = p\mu \tag{9}$$

where  $\mu$  is the Lagrange multiplier attached to the consumption expenditure constraint. Combining the first order conditions yields

$$\frac{1}{\chi} \frac{C_{NT}}{C_T} = p \tag{10}$$

Together with the consumption expenditure constraint we find the demand function for tradables and nontradables

$$C_{NT} = \frac{\chi}{1+\chi} \left(\frac{1}{P}\right)^{-1} C \tag{11}$$

$$C_T = \frac{1}{1+\chi} \left(\frac{p}{P}\right)^{-1} C \tag{12}$$

The demand for each good is proportional to real consumption C and depends on the ratio of the good's price (in terms of the numéraire) to the price index.

The foreign representative household is modelled in an equivalent way, being the owner of a foreign bank and facing a maximization problem identical to the home household's one. Variables in the foreign country will be denoted with an asterisk '\*'. The foreign household receives the same amounts of endowment and has the same intratemporal preferences as the home household. However, two completely symmetric countries, being equally endowed with and producing one single homogeneous good, have no reason to trade and hence, in such a set-up there would be no need for banks to intermediate international funds. As will become clear in section 2.3, our model only starts to be interesting at a point where the financial sector needs to absorb excess supplies of one of the two countries' bonds, i.e. when banks have an international portfolio. In order to induce a trade imbalance and hence an excess supply of the importing country's bonds we introduce one asymmetry between domestic and foreign households: We assume that  $\beta^* < \beta$ , i.e. that the foreign households have a relatively higher discount rate and are less patient than the home households. As a consequence, in such an otherwise symmetric set-up, the home country runs a trade surplus

in equilibrium.<sup>10</sup>

#### 2.2 Non-financial firms

The traded good is produced by perfectly competitive firms in the second period. The representative firm in the home country operates according to the following constant-returns-to-scale technology

$$Y_{T,1} = K_1^{\alpha} L_1^{1-\alpha} \tag{13}$$

Note that labor  $L_1$  and capital  $K_1$  are internationally immobile. In the first period, the firm can transform traded goods into capital and then use it for production in the second period. One unit of output invested raises capital by one unit. This process is reversible, so that a unit of capital, after having been used to produce output, can be retransformed into the tradable consumption good. The firm obtains the necessary funds for this investment by issuing claims  $S_0$  at price  $q_0$ . One claim finances one unit of capital, so we have

$$S_0 = I_0 = K_1 \tag{14}$$

$$q_0 = p_0 \tag{15}$$

Given our assumption on the capital transformation process, the price of capital (investment) is always equal to the price of output. From now on we will use  $p_t$  as the price per claim and refer to  $K_1$  as the total supply of claims.

The firm maximizes its profit by choosing optimal employment and investment, taking all prices as given:

$$\max_{K_1, L_1} p_1 Y_{T,1} - Z_1 K_1 - w_1 L_1$$
subject to (13)

where  $Z_1$  is the cost of capital to the firm, or the profit flow from a claim financing one unit of capital to the holder of this security, measured in terms of the domestic numéraire. The first order conditions are standard, we have

$$K_1: Z_1 = \alpha \left(\frac{L_1}{K_1}\right)^{1-\alpha} p_1 (16)$$

$$L_1: w_1 = (1 - \alpha) \left(\frac{K_1}{L_1}\right)^{\alpha} p_1 (17)$$

In period 1, after production the firm is left with  $(1 - \delta)K_1$  units of capital, which represents the

 $<sup>^{10}</sup>$ Obviously, there would be other ways to induce a trade imbalance across the two countries. One alternative would be to make the countries differ in their initial endowments of tradable goods,  $Y_{T,0}$  and  $Y_{T,0}^*$ . In a otherwise symmetric set-up, the country with the higher level of endowment would be the exporter in period 0. Another possibility involves heterogeneous production technologies. For example, a higher relative productivity in the second period in a otherwise symmetric set-up would make a country an importer in the first period.

outstanding claim of the security holders and is returned to them. Therefore, the rate of return on one home investment security is:

$$R_{k,1} = \frac{Z_1 + (1 - \delta)p_1}{p_0} \tag{18}$$

Foreign non-financial firms are modelled in an equivalent way, i.e., they face the same production technology as the home firms. It follows that the rate of return on one foreign investment security is

$$R_{k,1}^* = \frac{Z_1^* + (1-\delta)p_1^*}{p_0^*}, \quad \text{where} \quad Z_1^* = \alpha \left(\frac{L_1^*}{K_1^*}\right)^{1-\alpha} p_1^*$$
 (19)

By assumption, the law of one price holds, so

$$p_0 = e_0 p_0^* \tag{20}$$

$$p_1 = e_1 p_1^* \tag{21}$$

 $e_t$  is the real exchange rate, defined as the price of the foreign numéraire (i.e. the foreign non-traded good) in terms of home numéraire (i.e. the home non-traded good).<sup>11</sup>

#### 2.3 Banks

Banks are at the core of our model. We assume that financial markets are segmented, implying that non-financial agents cannot lend funds directly to each other. Banks act as the financial intermediaries in two types of financial markets: The investment market and the international credit market. In the former, banks intermediate funds between households and firms and in the latter between the agents of the two countries by absorbing imbalances resulting from trade flows (later we will also consider imbalances from financial flows). Due to an agency problem between creditors and banks, however, this financial intermediation is imperfect.

Imperfect financial intermediation is visible in the form of spreads or excess returns in the two financial markets. Due to the deterministic set up of our model, these spreads reflect arbitrage opportunities. One might now argue that in reality the foreign exchange market is huge and there are vast quantities of capital around such that it is unlikely to find arbitrage opportunities. However, also the biggest players face financial constraints depending on their risk bearing capacities and existing balance sheet risks. Moreover, as noted by, for example, Shleifer and Vishny (1997), arbitrage according to the basic textbook definition, i.e. involving neither capital nor risk, does hardly exist in practice. Arbitrageurs typically invest other people's money which induces an agency relationship. As a consequence, arbitrageurs only have limited access to funds. Furthermore, the investors providing the arbitrageurs with the funds are likely to be sensitive to the past performance of these arbitrageurs and might withdraw their money precisely when prices/exchange rates move

 $<sup>^{11}</sup>$ The relationship between  $e_t$  and the CPI exchange rate is derived in Appendix D.

against them and their participation is needed most. Hence, excess returns and limits to arbitrage are likely to arise in extreme situations.

In the real world, the observed excess returns in our two financial markets might to a large extent be related to risks. Excess returns in the international credit market, e.g., which are the rule rather than the exception, would correspond to a deviation from UIP. There is a broad agreement in the literature that the spreads in UIP do not necessarily correspond to arbitrage opportunities, but might reflect a fair compensation that investors demand for holding currency risk (for a review of the recent literature see Engel (2014)). In our model, banks are meant to capture the large players in the global financial markets like JP Morgan and Goldman Sachs, but also currency hedge funds as well as active investment managers and pension funds. As is stressed in Gabaix and Maggiori (2015), even though different in many aspects, all these institutions have in common that they often bear the ultimate risk which arises because households' demand is unbalanced. By taking the other side, they benefit from the medium-term imbalances stemming from imbalances in currency demand due to trade and financial flows. In this sense, the spreads in our model can also be interpreted as a compensation that these institutions demand for holding currency risk.

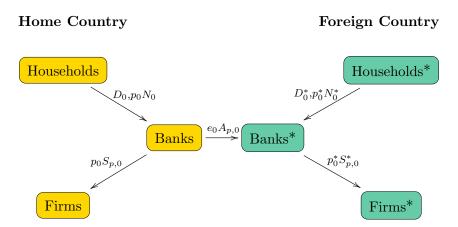


Figure 1: Banking sector: Period-0 flows.

Figure 1 gives an overview of the period-0 flows in the banking sector. In each country, there is a continuum of identical banks that have unit mass. The representative home and foreign bank are modelled in a similar way. Differences between the two result from their interaction on the international interbank market. This interaction in turn is a result of the trade (and later also portfolio) flows between the two countries. Since the domestic country runs a trade surplus in the first period, there is an excess supply of foreign bonds, i.e. claims on traded goods issued in the foreign country and denominated in the foreign numéraire. More precisely, a trade surplus in the home and, hence, a trade deficit in the foreign country imply that foreign households consume relatively more by bringing less of their endowment of traded goods to the foreign bank to save it  $(D_0 > D_0^*)$ . As a consequence, the foreign bank has relatively less traded goods available to invest in local firms. To increase these investments it issues foreign bonds to the home bank and receives

traded goods in exchange. This is how the domestic bank absorbs the excess supply of foreign bonds from the foreign country.

First, let us have a look at the home bank. The balance sheet of the home bank is

$$p_0 S_{p,0} + e_0 A_{p,0} = D_0 + p_0 N_0 (22)$$

In addition to obtaining funds from households by issuing domestic bonds  $D_0$ , the bank receives seed capital from its owner. This seed capital represents the bank's net worth in the first period,  $N_0 > 0$ .  $p_0 N_0$  may be thought of as the bank's equity capital and  $D_0$  as its debt. The bank uses these funds, on the one hand, to invest in domestic firms by buying home investment securities in the amount of  $p_0S_{p,0}$  on the domestic investment market. On the other hand, it uses them to purchase foreign bonds, issued by the foreign bank, in the amount of  $e_0A_{p,0}$  on the international credit market. All variables in the home bank's balance sheet, except for  $A_{p,0}$ , are measured in terms of the domestic numéraire.  $A_{p,0}$  is measured in terms of the foreign numéraire and therefore multiplied by the exchange rate  $e_0$ .

The interaction between the bank and the firm and hence the transfer of funds from banks to firms is frictionless. This, however does not hold for the interaction between the bank and its creditors. It is at this point that we introduce the key feature of our model: Financial frictions, which lead to imperfect financial intermediation and are endogenized by the introduction of a limited commitment problem. Following Gertler and Karadi (2013), we assume that in period 0, after taking positions, the bank can choose to divert a certain fraction of the assets it holds and transfer the proceeds to the household it is owned by. In particular, the bank can divert a fraction  $\theta$  of its holdings of claims on firms  $p_0S_{p,0}$  and a fraction  $\Delta$  of its foreign bond holdings  $e_0A_{p,0}$ . The creditors can react by forcing the bank into bankruptcy and claiming the remaining fraction of assets. An overview of the domestic bank's balance sheet and the divertable parts is provided in Table 1 (left-hand side).

	Home Bank			Foreign Bank	
	Assets	Liabilities		Assets	Liabilities
	$p_0S_{p,0}$	$D_0$		$p_0^* S_{p,0}^*$	$D_0^*$
$\theta$			$\theta^*$ {		$A_{p,0}$
					$p_0^* N_0^*$
$\Delta$ $\left\{  ight.$	$e_0 A_{p,0}$	$p_0N_0$			

**Table 1:** Balance sheet of the banks in period 0

As Gertler and Karadi (2013) we allow the friction parameters in the investment and the international credit market to differ  $\left(\Delta \leq \theta\right)$ .  $\Delta > \theta$ , for example, implies that it is easier for the bank to divert foreign bonds as compared to claims on firms. The intuition behind is that the performance of some assets in the bank's portfolio may be less transparent for creditors and therefore an easier target for diversion.

The bank has no incentive to misbehave if

$$V_0 \ge \theta p_0 S_{p,0} + \Delta e_0 A_{p,0} \tag{23}$$

where  $V_0$  is the net present value of the bank. Hence, the bank has no incentive to divert if its discounted profit, which is given up under diversion, is greater or equal to the gain from diversion.

Creditors anticipate the possibility of diversion and will limit the amount of funds they lend to the bank. They will only buy domestic bonds, and hence the bank can only issue such bonds, as long as equation (23) is fulfilled. Thus, it represents the incentive constraint (IC) that the bank faces.

The discounted profit of the bank is

$$V_0 = \Lambda_{0,1} \left( R_{k,1} p_0 S_{p,0} + R_1^* \frac{e_1}{e_0} e_0 A_{p,0} - R_1 D_0 \right)$$
(24)

 $R_{k,1}$ ,  $R_1^*$  and  $R_1$  is the gross return on home investment securities, on foreign bonds and on domestic bonds, respectively. Since the household is the bank's owner, the period-1 profit is discounted by the household's intertemporal marginal rate of substitution  $\Lambda_{0,1}$ . Using balance sheet equation (22) we can rewrite the discounted profit in terms of excess returns  $(R_{k,1} - R_1)$  and  $\left(R_1^* \frac{e_1}{e_0} - R_1\right)^{12}$ :

$$V_0 = \Lambda_{0,1} \left( \left( R_{k,1} - R_1 \right) p_0 S_{p,0} + \left( R_1^* \frac{e_1}{e_0} - R_1 \right) e_0 A_{p,0} + R_1 p_0 N_0 \right)$$
(25)

Note that  $V_0 = \Lambda_{0,1} p_1 N_1$ , where  $p_1 N_1$  is the value of the equity capital in period 1. The optimization problem of the bank is

$$\max_{S_{p,0}, A_{p,0}} V_0$$
subject to (23), (25)

The first order conditions yield

$$S_{p,0}: \qquad \Lambda_{0,1} (R_{k,1} - R_1) = \frac{\lambda}{1+\lambda} \theta$$
 (26)

$$A_{p,0}: \qquad \Lambda_{0,1}\left(R_1^* \frac{e_1}{e_0} - R_1\right) = \frac{\lambda}{1+\lambda}\Delta$$
 (27)

<sup>&</sup>lt;sup>12</sup>As in Gertler and Karadi (2013), when referring to "excess return" we mean the difference between the actual return and its value under frictionless markets, while in finance the term is used to reflect the premium due to risk (within a frictionless set-up). In the context of the international credit market, and as mentioned above, we furthermore interpret the term excess return as a risk/safety premium and therefore use these words interchangeably. Herewith, we follow Gabaix and Maggiori (2015) and refer to the spread arising in the interest rate parity as a risk premium even tough it does not stem from uncertainty and is therefore not a risk premium in the traditional sense. The term seems to be justified, however, when thinking about the empirical literature, where it is common to give any excess returns observed the label risk premium, no matter whether they stem from uncertainty or from some other kind of imperfection. Moreover, Gabaix and Maggiori (2015) illustrate a straightforward way to, in a model that involves uncertainty, make the spread directly depend on the amount of risk.

where  $\lambda$  is the Lagrange multiplier attached to the incentive constraint.

If the financial friction parameters  $(\theta, \Delta)$  are small, the IC is not binding and  $\lambda = 0$ . Intuitively, for very low values of  $\theta$ ,  $\theta^*$  or  $\Delta$ , respectively, the divertable part of a bank's assets will inevitably be lower than the bank's equity capital, which it would loose in case of misbehaviour. Thus, the banks' incentive constraint will not be binding and we are in a frictionless environment, where banks acquire assets up to the point where no arbitrage possibilities are left and excess returns are zero. Firms can borrow at the home interest rate  $R_{k,1} = R_1$  and the interest rate parity (IP)  $R_1 = R_1^* \frac{e_1}{e_0}$  holds.

If  $\theta$  and/or  $\Delta$  are above a certain threshold, then the IC is binding and  $\lambda > 0$ . We are in a model environment of financial frictions that become visible in the form of positive excess returns in the respective market(s). Compared to the frictionless equilibrium there is less financial intermediation. The bank would like to borrow more funds and invest them in the respective market(s) to earn the excess return (arbitrage), however, creditors are unwilling to provide the bank with more funds. These limits to arbitrage lead to higher returns on claims on firms in the home investment market,  $R_{k,1} > R_1$ , and to a deviation from the IP in the international credit market,  $R_1^* \frac{e_1}{e_0} > R_1$ . As mentioned above, the latter can be interpreted as a premium that the home bank requires in order to be willing to absorb the imbalances in the international credit market and not divert any of their assets. Moreover, the size of this spread reflects the risk of shifting funds from the home to the foreign country. In this sense, the exchange rate change between periods 0 and 1 incorporates a risk premium on foreign bonds or safety premium on domestic bonds, respectively.<sup>13</sup> The higher  $\lambda$ , the tighter the IC is binding and the higher is or are the excess return(s).

Notice that equations (26) and (27) imply the following no-arbitrage relation:

$$(R_{k,1} - R_1) = \frac{\theta}{\Delta} \left( R_1^* \frac{e_1}{e_0} - R_1 \right) \tag{28}$$

If  $\theta$  is smaller than  $\Delta$ , the excess return on home investment securities is equal to a fraction  $\frac{\theta}{\Delta}$  of the excess return on foreign bonds. This occurs because in that case, the proportion of funds that the home bank can divert from their investment portfolio is only a fraction  $\frac{\theta}{\Delta}$  of the proportion they can divert from their foreign bonds portfolio. As a result, limits to arbitrage are weaker for home securities than for foreign assets. Obviously, if  $\theta$  is larger than  $\Delta$ , just the opposite holds true.

The households' willingness to lend and hence the size of the bank's portfolio does not only depend on the fractions that the bank can divert, but also on the size of the bank's equity capital. The limited commitment of the bank generates an endogenous capital constraint (CC). One can see the restriction that the IC places on the size of the bank's portfolio relative to its net worth

<sup>&</sup>lt;sup>13</sup>For the sake of simplicity, our model does not involve uncertainty. Hence, as mentioned before, and by construction, there are riskless arbitrage opportunities. However, it would be straightforward to make it more realistic and introduce general uncertainty about the future, i.e. period 1, and herewith make these riskless arbitrage opportunities disappear by allowing banks to eliminate them. In particular, in the international credit market, one could then argue that the Covered Interest Parity (CIP) is always satisfied, while disruptions in financial intermediation lead to deviations from the Uncovered Interest Parity (UIP).

when the IC is rewritten as follows (using the first order conditions and equation (25)):

$$CC = \begin{cases} \frac{\Delta \Lambda_{0,1} R_1}{\Delta - \Lambda_{0,1} \left( R_1^* \frac{e_1}{e_0} - R_1 \right)} p_0 N_0 \ge \theta p_0 S_{p,0} + \Delta e_0 A_{p,0} & \text{if} & \theta \ge 0, \Delta > 0 \\ \frac{\theta \Lambda_{0,1} R_1}{\theta - \Lambda_{0,1} \left( R_{k,1} - R_1 \right)} p_0 N_0 \ge \theta p_0 S_{p,0} + \Delta e_0 A_{p,0} & \text{if} & \theta > 0, \Delta \ge 0 \\ \text{no CC} & \text{if} & \theta = 0, \Delta = 0 \end{cases}$$
(29)

The exact derivation of capital constraint (29) is provided in Appendix A. On the left-hand side, we have the bank's net worth multiplied by some weight. On the right-hand side, we have a measure of the bank's portfolio. The weights  $\theta$  and  $\Delta$  represent the weaker (stronger) limits to arbitrage in the investment market if  $\theta < \Delta$  ( $\theta > \Delta$ ), which allows the bank to acquire a relatively higher (lower) portfolio share of claims on firms compared to foreign bonds. The higher  $N_0$  the more assets the bank can acquire and the larger is its portfolio size. Thus, capital constraint (29) reveals that a high  $\Delta$ , meaning that the bank can divert a high amount of its foreign assets, stands for a low ability of this bank to intermediate international funds, which in the aggregate reflects a disruption in the international credit market. Likewise, a high  $\theta$  stands for the bank to be only limitedly capable of intermediating investment funds. In the aggregate, this reflects a disruption in the home investment market. In this respect, we can interpret the divertable fractions  $\theta$  and  $\Delta$  also as a measure for the risk bearing capacity of the bank. The higher they are, the lower is the banks' risk bearing capacity and the less funds they can intermediate in the respective markets.

The set-up of the foreign bank is similar to the one of the domestic bank with some differences in the balance sheet reflecting and resulting from the interaction of the two on the international credit market. The balance sheet of the foreign bank is (see Table 1, right-hand side)

$$p_0^* S_{p,0}^* = D_0^* + A_{p,0} + p_0^* N_0^*$$
(30)

The foreign bank can divert a fraction  $\theta^*$  of its holdings of claims on firms  $p_0^* S_{p,0}^*$  and faces the following incentive constraint

$$V_0^* \ge \theta^* p_0^* S_{p,0}^* \tag{31}$$

The discounted profit of the foreign bank is

$$V_0^* = \Lambda_{0,1}^* \left( R_{k,1}^* p_0^* S_{p,0}^* - R_1^* D_0^* - R_1^* A_{p,0} \right)$$
  

$$\Leftrightarrow V_0^* = \Lambda_{0,1}^* \left( (R_{k,1}^* - R_1^*) p_0^* S_{p,0}^* + R_1^* p_0^* N_0^* \right)$$
(32)

The optimization problem is

$$\max_{S_{p,0}^*} V_0^*$$
 subject to (31), (32)

which yields the following first order condition

$$S_{p,0}^*: \qquad \Lambda_{0,1}^* \left( R_{k,1}^* - R_1^* \right) = \frac{\lambda^*}{1 + \lambda^*} \theta^*$$
 (33)

The restriction on the foreign bank's portfolio, i.e., the foreign bank's endogenous capital constraint, is

$$CC^* = \begin{cases} \frac{1}{\theta^* - \frac{1}{R_1^*} (R_{k,1}^* - R_1^*)} p_0^* N_0^* \ge p_0^* S_{p,0}^* & \text{if } \theta^* > 0\\ \text{no } CC^* & \text{if } \theta^* = 0 \end{cases}$$
(34)

Note that we chose to set up the banking sector in a way that allows to look at the home country as the net exporter.<sup>14</sup> This set-up is only valid when there is a non-negative excess supply of foreign bonds (abstracting from central bank interventions), or more generally, when the amount of foreign bonds that need to be absorbed by the home banks is larger or equal to zero. The assumption that the foreign households have a relatively higher discount rate than the home households ( $\beta^* < \beta$ ) ensures that in the frictionless case and hence also for a positive range of friction parameter values, this always holds true.

#### 2.4 Market Clearing

To close the model, we require the markets for assets, labor and goods to clear. Therefore, in the home and foreign investment markets as well as in the markets for home and foreign bonds, it must hold that

$$S_{p,0} = K_1 (35)$$

$$S_{p,0}^* = K_1^* \tag{36}$$

$$B_0 = D_0 \tag{37}$$

$$B_0^* = D_0^* \tag{38}$$

Remember that  $K_1$  and  $K_1^*$  are total supplies of investment securities. In the labor market, labor demand in each country needs to equal the inelastic labor supply

$$L_1 = L \tag{39}$$

$$L_1^* = L^* \tag{40}$$

In the goods markets, the market clearing condition for traded goods requires that

$$Y_{T,0} + Y_{T,0}^* = C_{T,0} + C_{T,0}^* + K_1 + K_1^*$$
(41)

$$Y_{T,1} + Y_{T,1}^* + (1 - \delta)K_1 + (1 - \delta)K_1^* = C_{T,1} + C_{T,1}^*$$
(42)

<sup>&</sup>lt;sup>14</sup>An analysis from the perspective of a deficit country can be made by looking at the foreign country.

Finally, for simplicity, we assume that the endowment of nontraded goods is constant across countries and time:  $Y_{NT,t} = Y_{NT,t}^* = \chi$ . Hence, for the nontraded goods it must hold that

$$C_{NT,0} = \chi \tag{43}$$

$$C_{NT.1} = \chi \tag{44}$$

$$C_{NT,0}^* = \chi \tag{45}$$

$$C_{NT,1}^* = \chi \tag{46}$$

Note that combining the budget constraints of the home households and the home banks in period 0 and using the market clearing condition for domestic bonds, claims on domestic firms and nontraded goods yields the market clearing equation that describes the equilibrium on the international credit market:

$$e_0 A_{p,0} = p_0 \underbrace{(Y_{T,0} - K_1 - C_{T,0})}_{\equiv NX_0}$$
(47)

where the right-hand side of the above equation is equal to  $p_0$  times the net exports of the home country in the first period  $NX_0$ .<sup>15</sup> Equilibrium on the international credit market requires that the excess supply of foreign bonds in the amount of  $p_0NX_0$  is all absorbed by home banks and therefore equal to their demand for foreign bonds  $e_0A_{p,0}$ .

Even tough this is a parsimonious model there exists no closed-form solution. However, we can solve the system of equations numerically. A summary of the equilibrium conditions is provided in Appendix A.

<sup>&</sup>lt;sup>15</sup>Note that the home country's net exports in the second period are  $NX_1 = -R_1^* \frac{e_1}{e_0} \frac{p_0}{p_1} NX_0$ .

# 3 Frictionless Equilibrium

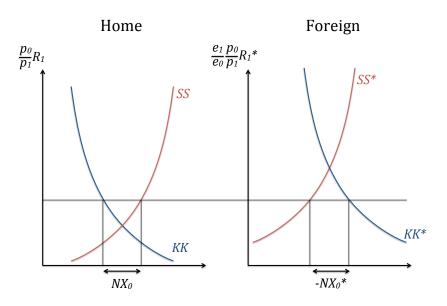


Figure 2: Frictionless equilibrium.

In order to get a better understanding of the model it is useful to first have a look at its solution in a frictionless world. But before doing so, note that plugging the market clearing condition for the non-tradable good into demand equation (11) reveals the general result that total consumption expenditure is constant over time  $(P_0C_0 = P_1C_1)$ , from which in turn it follows that the rate of return on domestic bonds must be equal to the inverse of the time preference parameter:  $R_1 = 1/\beta$  (see equation (7)). Likewise, the rate of return on foreign bonds is  $R_1^* = 1/\beta^*$ .

In a frictionless environment, the parameters  $\theta$ ,  $\Delta$  and  $\theta^*$  are either equal to zero or sufficiently low for the banks' incentive constraint not to be binding. Banks face no limits to arbitrage, hence excess returns are zero in all market and returns when measured in the domestic numéraire are equalized, i.e.  $R_1 = R_{k,1} = R_1^* \frac{e_1}{e_0} = R_{k,1}^* \frac{e_1}{e_0}$ . A home appreciation from period 0 to period 1 makes up for the higher rate of return on foreign bonds caused by the foreign households' impatience:  $e_1 = \frac{\beta^*}{\beta} e_0$ . Given that domestic and foreign non-financial firms have the same production technology and under the condition that the labor forces and the levels of bank equity capital in the two countries are of equal size, equalization of returns implies that investment in capital and hence production in the second period is the same in both countries:  $K_1 = K_1^*$  and  $Y_{T,1} = Y_{T,1}^*$ . However, as the home households are more patient, they save relatively more than the foreign households and have a lower level of first-period and a higher level of second-period consumption compared to the foreign households, which is reflected in positive net exports of the home country in period 0.

Graphically, the world equilibrium can be illustrated in a Metzler diagram. Figure 2 illustrates first-period saving  $(S_0 = Y_{T,0} - C_{T,0})$  and  $S_0^* = Y_{T,0}^* - C_{T,0}^*$  and investment  $(I_0 = K_1)$  and  $I_0^* = K_1^*$  schedules for the home and the foreign country, with the two country's real interest rates  $\frac{p_0}{p_1}R_1$  and

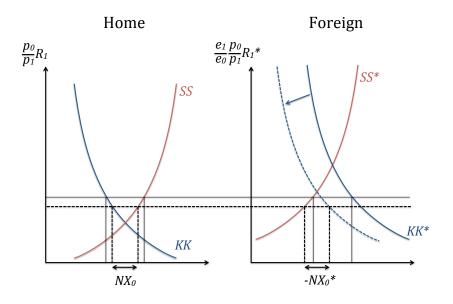
 $\frac{p_0^*}{p_1^*}R_1^* = \frac{e_1}{e_0}\frac{p_0}{p_1}R_1^*$  on the vertical axis.<sup>16</sup> A formal definition of the investment schedules (KK) and  $(KK^*)$  and the savings schedules (SS) and  $(SS^*)$  can be found in Appendix E. In the frictionless equilibrium, the real interest rates in the two countries are equal and equate global saving to global investment. Net exports of the home country have to correspond to net imports of the foreign country. An analytical solution of the frictionless model is provided in appendix B.

#### 4 The Role of Financial Frictions

As stated already, it is not possible to solve the model under financial frictions analytically. Graphical and numerical illustration in this chapter shall help to see the implications of the different frictions and to get intuition about the mechanisms at work.

Before turning to the description of the implications of financial frictions in our model, remember that, as derived in section 3, the bond market interest rates are  $R_1 = 1/\beta$  and  $R_1^* = 1/\beta^*$ . This result holds independently of whether financial markets exhibit limits to arbitrage or not and will prove useful in the following sections. It follows that any movements in excess returns must come from a change in  $R_{k,1}$ ,  $R_{k,1}^*$  or  $\frac{e_1}{e_0}$ , respectively. In particular, this implies that any change in the interest parity must come from a change in the rate of appreciation over the two periods.

#### 4.1 Effect of Financial Frictions in the Foreign Investment Market



**Figure 3:** Financial frictions in the foreign investment market:  $\theta^* > 0$ . The solid lines represent the frictionless equilibrium, the dashed lines the equilibrium with the friction.

<sup>&</sup>lt;sup>16</sup>We use the term *real* to mean in terms of the traded good. It is this real rate of return that matters for the households' consumption and savings decision. Due to the law of one price, we have:  $\frac{p_0^*}{p_1^*}R_1^* = \frac{e_1}{e_0}\frac{p_0}{p_1}R_1$ .

Consider first the effect of financial frictions in the foreign investment market, captured by an increase in the foreign investment market friction parameter  $\theta^*$ , with  $\theta$  and  $\Delta$  set to zero. When  $\theta^*$  is sufficiently large for the foreign banks' incentive constraint and hence also the endogenous capital constraint to become binding ( $\lambda^* > 0$ ), foreign banks are hindered to exploit all arbitrage opportunities and excess returns on the foreign investment market become positive:  $R_{k,1}^* - R_1^* > 0$  (see equation (33)). Excess returns on the home investment market and the international credit market, on the other hand, remain zero (see equations (26) and (27)). Combining the foreign banks' capital constraint (34) and the foreign investment market clearing condition (36) reveals that with the constraint starting to be binding, the level of capital in the foreign country will obviously be limited:

$$\frac{1}{\theta^* - \frac{1}{R_1^*} \left( R_{k,1}^* - R_1^* \right)} p_0^* N_0^* \ge p_0^* K_1^* \tag{48}$$

(As  $\lambda^* > 0$ , this equation will hold with equality.)

Graphically, financial frictions in the foreign investment market shifts the foreign investment curve to the left (see Figure 3)<sup>17</sup>. For a given real rate of return  $\frac{e_1}{e_0} \frac{p_0}{p_1} R_1^*$ , investment in the foreign country decreases as the foreign banks' ability to intermediate funds in this market has decreased and they face limits to arbitrage. Costs of capital in the foreign market increase. In order to maintain the world equilibrium, the equilibrium real rate of return has to decrease. Due to the frictions in the foreign investment market, the home country will in equilibrium on the one hand decrease its savings, and on the other hand invest a larger part of its savings domestically, which causes net exports of the home country to decrease. The foreign country, on the other hand, will in equilibrium also decrease its savings, but at the same time reduce investments to a larger extent, which altogether leads to a decrease in its net imports. Overall, there is a decrease in world savings and, consequently, world investments, implying a lower level of world output in the second period. Furthermore, the frictions in the foreign investment market lead to a misallocation of capital: While in the frictionless case, identical production technologies and equal size of the labor forces L and  $L^*$  imply that the level of investment is equalized across the two countries, now a majority of capital is invested in the home country. This change in the allocation implies that relative to the frictionless level, the home country's output in the second period will increase while the foreign country's output in the second period will decrease, implying that there is a change in the two countries' fundamentals. The relative increase in the home country's lifetime resources 18 induces a home appreciation in both periods.

Figure 4 provides a numerical illustration of the effects of financial frictions in the foreign investment market that have just been described. Setting  $\theta$  and  $\Delta$  equal to zero and using the calibration

$$C_{NT,0} + p_0 C_{T,0} + \frac{1}{R_1} (C_{NT,1} + p_1 C_{T,1}) = Y_{NT,0} + p_0 Y_{T,0} + \frac{1}{R_1} (Y_{NT,1} + w_1 L + p_1 N_1) - p_0 N_0$$

<sup>&</sup>lt;sup>17</sup>For a formal proof of how an increase in  $\theta^*$  affects the two countries' saving and investment schedules, see Appendix E.

<sup>&</sup>lt;sup>18</sup>The home country's lifetime resources are apparent in the home households' intertemporal budget constraint:

of Table A.1 in Appendix A for the remaining parameters, it shows how the model's equilibrium evolves as the foreign investment market friction parameter  $\theta^*$  steadily increases, starting from a frictionless point. For small values of  $\theta^*$ , the foreign banks' incentive constraint is not binding. However, as soon as  $\theta^*$  is above a certain threshold, it does become binding and the equilibrium adjusts as described in the graphical analysis above. Any further increase in  $\theta^*$  has the same effects, i.e. it results in an additional home appreciation and another increase in excess returns in the foreign investment market.

The effects of financial frictions in the home investment market, captured by an increase in  $\theta$ , are symmetric to the one just described. When the home investment market friction parameter is sufficiently large for the home inventive constraint to become binding, the home investment curve shifts to the left resulting in positive excess returns in the home investment market and a home depreciation in both periods. A detailed analysis and the corresponding graphical and numerical illustration are provided in Appendix A.

The left-hand side shows the usage of these resources for intra- and intertemporal consumption, while the right hand-side indicates their origin. As the endowment of nontraded goods and period-0 traded goods is symmetric across countries and the law of one price holds, any relative changes of lifetime resources between the two countries can only come from relative changes in labor income  $w_1L = (1-\alpha)p_1Y_{T,1}$  or changes in the payoff of equity capital  $p_1N_1 = (R_{k,1} - R_1) p_0S_{p,0} + \left(R_1^* \frac{e_1}{e_0} - R_1\right) e_0A_{p,0} + R_1p_0N_0$ .

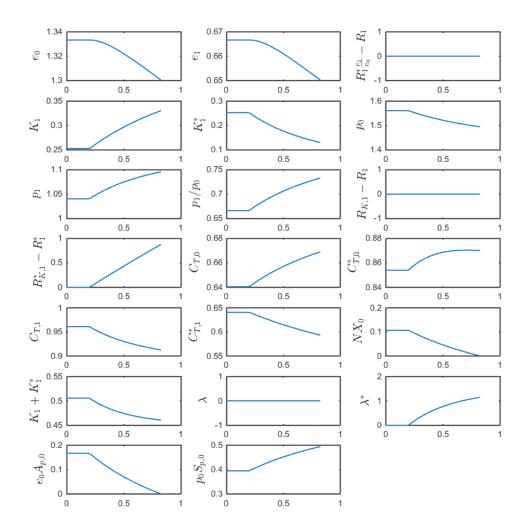


Figure 4: Effect of an increase in  $\theta^*$  on different variables ( $\theta^*$  on x-axis). Evolution of the model's equilibrium as the friction parameter in the foreign investment market increases starting from a frictionless point.  $\theta = 0$ ,  $\theta^* \ge 0$ ,  $\Delta = 0$ , the remaining parameter values are summarized in Table A.1 in Appendix A. Given that the set-up of the banking sector is only valid when  $e_0 A_{p,0} \ge 0$ , the plots only cover a limited range of possible values for  $\theta^*$ .

#### 4.2 Effect of Financial Frictions in the International Credit Market

Next, Figure 5 depicts the effect of financial frictions in the international credit market, captured by an increase in the international credit market friction parameter  $\Delta$ , with  $\theta$  and  $\theta^*$  set to zero. Hence, excess returns in the two investment markets remain zero (see equations (26) and (33)) and this type of frictions has no effect on the investment curves as the banks' ability to intermediate funds in their domestic markets remains unchanged. There is, however, a slight leftward shift in the home country's savings curve as home households increase their first-period consumption given that

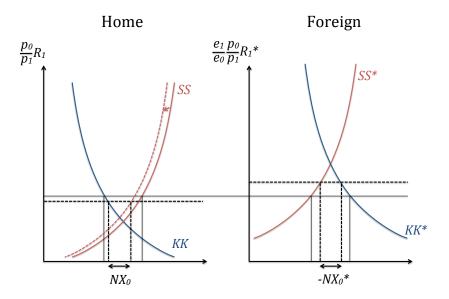


Figure 5: Financial frictions in the international credit market:  $\Delta > 0$ . The solid lines represent the frictionless equilibrium, the dashed lines the equilibrium with the friction.

they can expect a positive return on the home banks' equity capital they are holding. Due to the home banks' limits to arbitrage, the excess return on foreign bonds is positive and the home banks make positive profit. 19 The foreign country's saving curve does not move as in contrast to the home banks' portfolio, the foreign banks' portfolio is not international but purely domestic and therefore the foreign banks' profit and the return on the foreign households' equity capital holdings remain zero. As the home banks' ability to intermediate international funds decreases, their endogenous capital constraint becomes binding ( $\lambda > 0$ ) and they face limits to arbitrage in the international credit market, meaning that they can invest less funds in foreign bonds which results in a spread in the interest rate parity:  $R_{1e_0}^* - R_1 > 0$  (see equation (27)). The equilibrium real rate of return will now be higher in the foreign country than in the home country  $(\frac{e_1}{e_0}\frac{p_0}{p_1}R_1^* > \frac{p_0}{p_1}R_1)$ , representing the mentioned arbitrage opportunity which cannot be exploited by the home banks. Compared to the frictionless case, a larger part of the home country's first-period endowment is on the one hand consumed already in period 0 and on the other hand invested domestically. The credit constraint with respect to investments in foreign bonds makes the home banks reallocate their portfolio and invest a larger part in home investment securities. Altogether, this causes a decrease in the home country's net exports, as can also be derived analytically. Combining the equilibrium condition on the international credit market (47) and the home banks' capital constraint (29) shows how the

<sup>&</sup>lt;sup>19</sup>In the case of an increase in the investment market friction parameters  $\theta$  and  $\theta^*$ , there is no such shift in the savings curve of the respective country as the higher return on the households' holdings of equity capital (in this case due to the increase in the spreads  $R_{k,1} - R_1$  and  $R_{k,1}^* - R_1^*$ , respectively), is just nullified by a decrease in their second-period labour income due to the lower level of capital. For a formal proof of how an increase in  $\Delta$  affects the two countries' saving and investment schedules, see Appendix E.

amount of net exports is limited:

$$\frac{1}{\Delta - \frac{1}{R_1} \left( R_1^* \frac{e_1}{e_0} - R_1 \right)} p_0 N_0 \ge p_0 N X_0 \tag{49}$$

(As  $\lambda > 0$ , this equation will hold with equality.) Due to their lower risk bearing capacity, the home banks are less able to absorb excess supplies of foreign bonds, which then obviously goes hand in hand with a reduction in international trade. In the end, the frictions in the international credit market makes intertemporal trade more costly. It works like a tax on capital flows and makes the current account gap shrink.<sup>20</sup> The foreign country, on the other hand, receives less international funds, which in turn leads to a reduction in the amount of funds available for investment in foreign capital which is only partially compensated by the increase in the foreign households' savings.

In contrast to the frictions in the investment markets, the frictions in the international credit market do not directly limit the banks' capacity to intermediate funds between households and firms, but limit the international mobility of funds, which then indirectly leads to a misallocation of capital. The levels of investment are not equal as in the frictionless case, instead, a majority of capital is invested in the home country. There are now two sources of home appreciation in period 0. First, there is the increase in the home country's relative lifetime resources coming from the change in the allocation of capital, which implies that relative to the frictionless level, the home country's output in the second period will increase and the foreign country's output in the second period will decrease, implying that there is a change in fundamentals which induces a home appreciation in both periods. Second, the frictions in the international credit market result in the mentioned spread in the interest rate parity or safety premium on domestic bonds, i.e. a lower foreign depreciation from period 0 to period 1, and requires a home appreciation in the first (and a depreciation in the second) period.

Figure 6 provides a numerical illustration of these results. With  $\theta$  and  $\theta^*$  set to zero, it shows the evolution of the model's equilibrium as the international credit market friction parameter  $\Delta$  steadily increases, starting from a frictionless point. For small values of  $\Delta$  the home banks' IC is not binding. However, as soon as  $\Delta$  is above a certain threshold, the IC becomes binding and the equilibrium adjusts as described in the graphical analysis above. Any further increase in  $\Delta$  has the same effects, i.e. it results in an additional home appreciation/depreciation in the first/second period, respectively, and another increase in the interest parity spread or safety premium.

 $<sup>^{20}</sup>$ Note that  $\Delta=1$  (i.e. banks can divert *all* foreign assets) does not imply that the current account gap is completely closed or that the banks do not hold foreign assets anymore (see also the numerical illustration in Figure 6): It just means that in case of misbehaviour, the banks could divert and keep the proceeds of all foreign assets. If, however, the excess returns they can earn on the foreign bonds when not diverting them are large enough, they still have no incentive to misbehave and the financial markets will work even with  $\Delta=1$ .

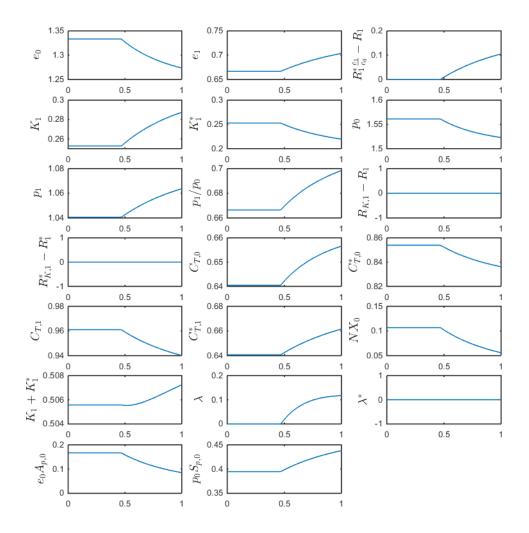


Figure 6: Effect of an increase in  $\Delta$  on different variables ( $\Delta$  on x-axis). Evolution of the model's equilibrium as the friction parameter in the international credit market increases starting from a frictionless point.  $\theta = 0$ ,  $\theta^* = 0$ ,  $\Delta \ge 0$ , the remaining parameter values are summarized in Table A.1.

#### 4.3 General Case

After studying separately the effects of financial frictions in one single market, we now turn to a description of the general model where all friction parameters are positive and banks in both countries face binding incentive constraints. In that case, excess returns are positive in all three markets, i.e. there is a spread in the interest rate parity and home as well as foreign firms face capital costs above the frictionless level. The banks' capital constraints (see equations (29) and (34)) combined with market clearing on the home and foreign investment markets as well as the international credit market (equations (35), (36) and (47)) describe how the levels of capital and

net exports are restricted:

$$\frac{1}{\Delta - \frac{1}{R_1} \left( R_1^* \frac{e_1}{e_0} - R_1 \right)} p_0 N_0 \ge \frac{\theta}{\Delta} p_0 K_1 + p_0 N X_0 \tag{50}$$

$$\frac{1}{\theta^* - \frac{1}{R_1^*} \left( R_{k,1}^* - R_1^* \right)} p_0^* N_0^* \ge p_0^* K_1^* \tag{51}$$

(As by assumption  $\lambda > 0$  and  $\lambda^* > 0$ , both equations will hold with equality.) In line with earlier reasoning, it can be seen from equation (50) that when limits to arbitrage are higher in the home investment market than in the international credit market (i.e. when  $\theta > \Delta$ ), intermediating capital in the home country is more constraining than intermediating net exports.

An interesting case to have a closer look at is the one where banks are equally constrained in all markets:  $\theta = \theta^* = \Delta > 0$ : From no-arbitrage relation (28), it follows that home banks will choose their portfolio such that excess returns in the domestic investment market and the international credit market are just equalized. Measured in terms of the home numéraire, we have  $R_1 < R_{k,1} = R_{1e_0}^* < R_{k,1e_0}^*$ , which implies that the level of investments is lower in the foreign country as compared to the home country. This results from the fact that home banks are constrained to hold less foreign bonds and hence intermediate less net exports relative to the frictionless case. Also note that the home banks' incentive constraint is more binding than the one of the foreign banks ( $\lambda > \lambda^*$ ). Intuitively, when banks are only constrained on the investment markets ( $\theta = \theta^* > 0$  and  $\Delta = 0$ ), the real interest rates in the two countries must be equalized and home and foreign banks hold the same amount in investment securities. Consequently, both incentive constraints are equally binding. Once  $\Delta$  is larger than zero, the restriction of the home banks increases, while tension on the foreign banks is released as less funds flow into the country and demand for intermediation falls. For a formal proof of these statements, see Appendix A.

Generally, a further increase in one of the friction parameters will make the banks shift funds away from the concerned market. A numerical illustration of the according effects is provided in Figures A.3 to A.5 in Appendix A. On the whole, the mechanisms are the same as the ones described in sections 4.1 and 4.2, even tough graphically (regarding the shifts in the saving and investment curves of the Metzler diagrams) there might be small deviations due to the non-zero Lagrange multipliers  $\lambda$  and  $\lambda^*$  and excess returns. Accordingly, a further increase in  $\theta^*$  will on the one hand lead to a decrease in the total amount of world investments  $(K_1 + K_1^*)$  and on the other hand to a shift of funds away from foreign investment securities and foreign capital towards home investment securities and home capital. Likewise, a further increase in  $\Delta$  will lead to a shift of funds away from foreign assets and herewith net exports and foreign capital towards home assets and home capital.

Obviously, higher frictions in one of the markets lead to an immediate increase in excess returns in this specific market. The spreads in the other markets, on the other hand, are only affected marginally, and only due to the changes in the Lagrange multipliers  $\lambda$  and  $\lambda^*$  (shadow prices). Higher frictions in the foreign investment market, for example, will make the home banks' incentive

constraint less binding as the foreign banks issue less foreign bonds and, in other words, the supply of foreign assets decreases. The home banks that now need to absorb less of these have more of their constrained funds free to invest in home investment securities. So, overall, the constraint on the home banks slightly relaxes and accordingly excess returns on the home investment market and the international credit market decrease. The portfolio adjustments of the banks that lead to a change in the allocation of capital, on the other hand, do obviously change the returns to capital. It is due to price adjustments (adjustments in  $\frac{p_0}{p_1}$ ) that the returns on investment securities (which are measured in terms of the numéraire, hence in terms of non-traded goods) hardly change.

## 5 Impact of International Portfolio Flows

So far, we have assumed that households are only able to trade bonds denominated in the numéraire of their own country, and that the imbalances in the demand for domestic and foreign bonds absorbed by the home banks have resulted from trade flows only. In this section, we introduce financial flows other than the ones resulting from trade imbalances and refer to them as international portfolio flows.<sup>21</sup> In particular, we now allow the home households to have an inelastic demand for foreign bonds f funded by an offsetting position  $e_0f$  in domestic bonds and the foreign households to have an exogenous inelastic demand for domestic bonds  $f^*$  funded by an offsetting position  $f^*/e_0$  in foreign bonds. We take the portfolio flows to be completely exogenous for the sake of simplicity and in order to avoid mixing up different channels. Hence, one could think of f and  $f^*$  as the result of simple noise or liquidity trading. In reality, however, a large part of the demand for foreign bonds is likely to be endogenous and depend on present and expected future fundamentals and, in particular, the interest rate differential. Thus, it would be more realistic to have  $f = f(R_1, R_1^*, e_0, e_1, \ldots)$  and  $f^*(R_1, R_1^*, e_0, e_1, \ldots)$ .<sup>22</sup> However, given that our main goal in this section is to increase understanding of the effects of international financial flows, but not the origin of these, we abstract from such dependencies.

The households' potential counterparties for financial transactions now not only comprise their domestic banks, but also the other country's banks. More precisely, home and foreign banks will now also issue bonds (in their own country's numéraire) to the other country's households and therefore have additional liabilities in the amount of  $f^*$  and f, respectively. Accordingly, the balance sheet equation of the home banks changes to:

$$p_0 S_{p,0} + e_0 A_{p,0} = D_0 - e_0 f + f^* + p_0 N_0$$
(52)

These portfolio flows alter equilibrium condition (47) on the international credit market which requires that net capital outflows have to be equal to net exports, or, put differently, that demand

<sup>&</sup>lt;sup>21</sup>Note that when talking about international portfolio flows we do *not* refer to portfolio investment in the sense as it is used in the Balance of Payments terminology.

<sup>&</sup>lt;sup>22</sup>For instance, as suggested in Gabaix and Maggiori (2015), a straightforward way to model a popular trading strategy, the carry trade, would be to set  $f = a + b(R_1 - R_1^*)$  and  $f^* = c + d(R_1 - R_1^*)$  for some constants a, b, c and d.

for foreign bonds has to equal supply of foreign bonds<sup>2324</sup>:

$$\underbrace{e_0 A_{p,0} + e_0 f}_{\text{gross}} - \underbrace{f^*}_{\text{gross}} = p_0 N X_0$$

$$\underbrace{e_0 A_{p,0} + e_0 f}_{\text{capital}} - \underbrace{f^*}_{\text{capital}} = p_0 N X_0$$

$$\underbrace{e_0 A_{p,0} + e_0 f}_{\text{gross}} - \underbrace{f^*}_{\text{gross}} = p_0 N X_0$$

$$\underbrace{e_0 A_{p,0} + e_0 f}_{\text{gross}} - \underbrace{f^*}_{\text{capital}} = p_0 N X_0$$

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$$\underbrace{e_0 A_{p,0} + e_0 f}_{\text{gross}} - \underbrace{f^*}_{\text{gross}} = p_0 N X_0$$

The home banks now have to absorb imbalances in the demand for foreign bonds stemming from trade and portfolio flows. Net demand for foreign bonds, given by the amount of net capital outflows, depends negatively on the amount of gross capital inflows  $f^*$ . Hence, the larger these inflows, the higher the amount of international funds that the home banks need to intermediate.

When the international credit market frictions parameter  $\Delta$  is equal to zero, the home banks are able to absorb any imbalances on the international credit market, that is, the interest rate parity holds and gross capital flows have no effect on any variable other than  $A_{p,0}$ . Consider the example of foreign households suddenly wanting to hold a certain amount of domestic bonds  $f^*$ . When banks are not constrained on the international credit market and the returns on domestic and foreign bonds are equalized, the home banks are willing to issue any additional amount of domestic bonds and in return increase their holdings of foreign bonds correspondingly. Hence, the home banks increase their holdings of foreign bonds  $e_0 A_{p,0}$  one to one with the inflow of capital  $f^*$ . So it is a trade that concerns but the foreign households and the home banks and does not affect the rest of the economy, i.e. the right-hand side of equation (53) stays constant. This irrelevance of gross capital flows is a common feature of the traditional international economics literature inspired by Dornbusch (1976) and Obstfeld and Rogoff (1995), where interest rate parity (or more specifically, the uncovered interest rate parity) is often either directly assumed to hold or imposed in the process of first order linearization.

Gross capital flows start to matter once the international credit market friction parameter  $\Delta$  is positive and the home banks' incentive constraint is or starts to be binding. When banks are credit constrained on the international credit market, gross flows have an impact on the tightness of the capital constraint they are facing:

$$CC = \begin{cases} \frac{\Delta}{\Delta - \frac{1}{R_1} \left( R_1^* \frac{e_1}{e_0} - R_1 \right)} p_0 N_0 \ge \theta p_0 K_1 + \Delta (p_0 N X_0 - e_0 f + f^*) & \text{if} & \theta \ge 0, \Delta > 0 \\ \frac{\theta}{\theta - \frac{1}{R_1} \left( R_{k,1} - R_1 \right)} p_0 N_0 \ge \theta p_0 K_1 + \Delta (p_0 N X_0 - e_0 f + f^*) & \text{if} & \theta > 0, \Delta \ge 0 \\ \text{no CC} & \text{if} & \theta = 0, \Delta = 0 \end{cases}$$
(54)

<sup>&</sup>lt;sup>23</sup>This simple set-up only works conditional on  $e_0A_{p,0} = p_0NX_0 - e_0f + f^* \ge 0$ . If portfolio outflows f are too large relative to portfolio inflows  $f^*$ ,  $A_{p,0}$  and hence the according divertable fraction would become negative in equilibrium. Furthermore, if  $e_0f > D_0$ , i.e. the home households go short in domestic bonds, we additionally need to make the assumption that no agency friction applies to the intermediation of these bonds. Given that they are "riskless" (as opposed to foreign bonds which incorporate currency risk and investment securities which incorporate investment risk), this seems a reasonable assumption, and one that is also discussed in Gertler and Karadi (2013) in the context of reserve holdings which yield the same return as domestic bonds.

<sup>&</sup>lt;sup>24</sup>Appendix C provides an overview of the equilibrium equations when international portfolio flows (and central bank intermediation) are introduced to the baseline model in section 2.

The inequalities in (54) will hold with equality given that we are considering here the case where banks are balance sheet constrained. Note that the critical value of  $\Delta$  at which the constraint becomes binding is endogenous and depends negatively (positively) on capital inflows  $f^*$  (outflows  $e_0f$ ). If, for example,  $f^*$  is very high, i.e. the excess supply of foreign bonds that the home banks need to absorb increases by a large amount, already a relatively low  $\Delta$  can make the banks' incentive constraint become binding.

A positive capital inflow shock, captured by an increase in  $f^*$ , or a negative capital outflow shock, captured by a decrease in f, make the capital constraint more binding as the home banks need a large part of their risk bearing capacity to intermediate these flows. Consider again the example of foreign households suddenly wanting to hold a certain amount of bonds of the home country  $f^*$ . When the home banks are credit constrained on the international credit market, we are at a point where creditors are not willing to provide the home banks with more funds since the banks would invest these (at least partially) in foreign bonds which in turn would result in higher proceeds under diversion and hence a higher incentive to misbehave. Capital inflows from foreign households, however, represent an exogenous increase in the home banks' funds and a higher amount of foreign bonds that the home banks are obliged to intermediate in order to maintain equilibrium on the international credit market (see equation (53)). Due to the binding balance sheet constraint, home banks are only able to intermediate these exogenous capital inflows when they simultaneously can relax their capital constraint in some other way.

This can work through two possible channels. The first one consists of adjustments on the households' or creditors' side. Home households will find it optimal to increase their period-0 consumption and decrease their savings, i.e. provide the home banks with less funds on their part. This and the resulting decrease in net exports and hence in the demand for domestic bonds leads to a relaxation of the capital constraint. A second channel works via the home banks' portfolio adjustment towards home investment securities which results in an automatic decrease of net exports. The first channel always plays an important role. If, among those markets where the home banks are active participants, the international credit market is the only market with a positive frictions (i.e.  $\theta = 0$ ), the second channel plays a significant role as well. If, on the other hand, both the international credit market and the home investment market frictions are positive, substituting the intermediation of net exports by an increase in holdings of home investment securities will not necessarily lead to a release in the tension on the balance sheet constraint. The higher the investment market frictions, the less such substitution yields the necessary release in the tension, and hence the more the first channel is at work, i.e. the more households will increase their period-0 consumption and reduce the amount of funds they provide the banks with.

When it is only the international credit market friction parameter that is positive, a capital inflow shock has mostly the same (qualitative) effects as an increase in  $\Delta$ : While an increase in  $\Delta$  represents a direct reduction in the ability of the banks to intermediate international funds, a capital inflow shock implies that a large part of the banks' risk bearing capacity is absorbed by these exogenous flows and hence represents an indirect reduction in their ability to intermediate

international funds. Graphically, a capital inflow shock can be represented by a widening of the spread between the two dashed vertical lines in Figure 5, corresponding to a larger deviation from interest rate parity and a home appreciation in period 0. Accordingly, also the results to the numerical example correspond for the most part to the one of an increase in  $\Delta$ , as can be seen in Figure A.6 in Appendix C. The portfolio inflow shock increases the misallocation of capital that is potentially already present due to frictions in the international credit market. The higher the portfolio inflows that the home banks need to intermediate, the larger is the amount of funds that they find optimal to invest in the home country. The resulting increase of the home country's second-period output compared to the foreign country's second period output raises the home country's relative lifetime resources and leads to further home appreciation pressure in period 0.

Note that in Figure A.6, under the initial parameter constellation ( $\theta = \theta^* = 0$ ,  $\Delta = 1/4$  and  $f^* = 0$ ), the economy is in the frictionless state. As long as  $f^*$  is small, the capital inflows can be absorbed by the banks without any further implications, and the economy remains in the frictionless state. But as they exceed a certain amount, they make the IC become binding and lead to limits to arbitrage in the international credit market.

Finally, Figures 7 and 8 provide numerical illustrations of the case where also the home investment market and both the home and foreign investment markets, respectively, exhibit limits to arbitrage. Note that since we set  $\Delta = \theta$ , excess returns in the international credit and the home investment market are of equal size.<sup>25</sup> Also in these situations, exogenous capital inflows trigger an increase in the excess return in the international credit market involving an increase in the safety premium on domestic bonds, and herewith an appreciation in period 0. Note that now, the levels of capital  $K_1$  and  $K_1^*$  change but marginally. As the home banks are equally constrained on the home investment market and the international credit market, no portfolio adjustment through an increase in home investment holdings will take place. Variations in the level of investment are of second order only. Hence, also the two country's relative second-period output changes but marginally if at all. However, compared to the foreign households, the home households will have a relatively higher payoff from their equity capital. The implicit decrease in the banks' ability to intermediate funds both in the international credit as well as in the investment market drives excess returns in both of these markets up (see equation (28)). While excess returns in the foreign investment market remain at zero if  $\theta^* = 0$ , they are also affected, but only marginally, if  $\theta^*$  is high enough for

 $<sup>^{25}</sup>$ If, in addition to setting  $\Delta=\theta,\,\theta^*$  is small enough for the foreign IC not to be binding, there is no misallocation of capital, i.e. like in the frictionless equilibrium the level of investment is equalized across the two countries. This is observable in Figure 7 and follows from the fact that compared to the frictionless equilibrium the home banks reduce their intermediation of investment securities and foreign bonds to the same extent (in real terms). The latter leads to an equivalent reduction of funds available to the foreign banks and hence, the same decrease in foreign as compared to home capital. In this case, the relation between returns is  $R_1 < R_{k,1} = R_1^* \frac{e_1}{e_0} = R_{k,1}^* \frac{e_1}{e_0}$ , i.e. the returns to capital when measured in the home numéraire are equal. As soon as  $\theta^*$  is high enough for the foreign IC to be binding, the foreign banks do not only receive less funds, but they are also limited in the intermediation of investment securities leading to a relatively stronger decrease in foreign relative to home capital (see Figure 8). The relation between returns is now  $R_1 < R_{k,1} = R_1^* \frac{e_1}{e_0} < R_{k,1}^* \frac{e_1}{e_0}$ , where the higher returns to foreign capital reflects the misallocation of capital towards the home country.

the foreign IC to be binding.<sup>26</sup> Hence, also in these cases, there is an increase of the home country's relative lifetime resources which involves further home appreciation pressure in the first period.

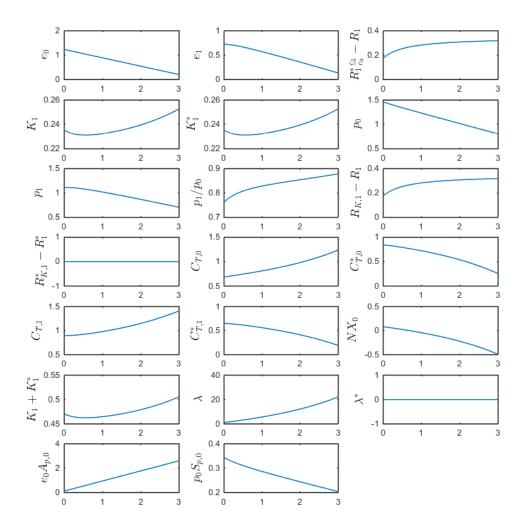


Figure 7: Effect of an increase in  $f^*$  on different variables ( $f^*$  on x-axis). Evolution of the model's equilibrium as capital inflows increase starting from a point where there are limits to arbitrage in the domestic investment market and the international credit market.  $\theta = 1/3$ ,  $\theta^* = 0$ ,  $\Delta = 1/3$ . The remaining parameter values are summarized in Table A.1.

<sup>&</sup>lt;sup>26</sup>This represents the main difference between Figure 7 and 8. If the foreign banks are constrained on the foreign investment market, the reduction in intermediated trade imbalances by the home banks affects the tightness of the binding foreign IC. Under low capital inflows, the foreign banks' liabilities decrease since the amount of foreign bonds borrowed from the home banks decreases more than the foreign household's savings increase. The foreign IC relaxes and excess returns slightly decrease. However, with increasing capital inflows the second effect dominates such that the foreign IC binds tighter and excess returns marginally increase.

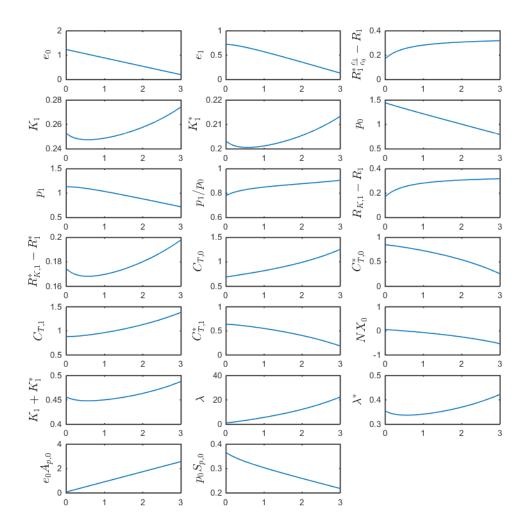


Figure 8: Effect of an increase in  $f^*$  on different variables ( $f^*$  on x-axis). Evolution of the model's equilibrium as capital inflows increase starting from a point where there are limits to arbitrage in all financial markets.  $\theta = 1/3$ ,  $\theta^* = 1/3$ ,  $\Delta = 1/3$ . The remaining parameter values are summarized in Table A.1.

### 6 Central Bank Asset Purchases

The previous sections have shed light on how frictions in financial markets - as they are experienced in the case of a financial crisis - and global imbalances in gross flows can lead to excess returns with negative consequences for investment and real activity as well as for international trade. We now take the perspective of the home country. While in this section, we show how large-scale asset purchases of the home central bank can generally help to reduce excess returns and thus mitigate the consequences of frictions in private intermediation and capital flow shocks, in the

next section, we are going to identify possible policy responses of this central bank to a variety of specific shocks. For this purpose, we define the goal of the central bank within the framework of our real model to bring the home country's economy closer to the frictionless state. This involves reducing spreads in the domestic credit market, but also stemming any "unjustified" appreciation pressure, i.e. appreciation pressure coming from increases in the safety premium on domestic bonds as opposed to appreciation pressure coming from a change in fundamentals only.

Our model allows to study and compare two policies that have repeatedly been put into practice in the course of the recent financial crisis, namely credit easing and foreign exchange interventions. By applying such policies the central bank can itself play the role of an intermediary, reduce the (excess) supply of investment securities and foreign bonds that needs to be absorbed by the private intermediaries and herewith relax the banks' capital constraint. In the context of our model, these two policy options are implemented as follows: The central bank can intervene in the domestic investment market by purchasing domestic claims on firms  $p_0S_{CB,0}$  and in the international credit market by purchasing foreign bonds  $e_0A_{CB,0}$ , issuing in both cases domestic bonds  $D_{CB,0}$  to finance these transactions.<sup>27</sup> Following the baseline scenario of Gertler and Karadi (2013), we assume here that the central bank issues these domestic bonds directly to the households. However, as is also discussed in Gertler and Karadi (2013),  $D_{CB,0}$  can as well be interpreted as reserves held by banks on account at the central bank. The equilibrium conditions are then identical to the ones in the baseline scenario under the assumption that no frictions apply to the intermediation of reserves.<sup>28</sup>

The crucial feature of the central bank is that as opposed to the private intermediaries it is not balance sheet constrained as it is not facing a limited commitment problem. We also make the simplifying assumption that both types of interventions, i.e. foreign exchange interventions and credit easing, are free of cost. This implies that in intermediating funds the central bank is equally efficient as private intermediaries. However, this assumption is not critical and is made for the sake of convenience. It would be straightforward to introduce relative efficiency costs as is done for example by Gertler and Karadi (2013) to capture the fact that a central bank is likely to be less efficient in intermediating foreign bonds and private securities than ordinary banks. For welfare considerations (from which we abstract in this paper), central bank interventions would then only be desirable when private intermediation is significantly constrained and even then only if efficiency costs were not too large. The latter, on the other hand, seems to be quite a reasonable assumption to make, so that in the end these costs would not change any of the qualitative implications of the model.

As a result of such interventions, the central bank's balance sheet, as described by the following equation, expands:

$$p_0 S_{CB,0} + e_0 A_{CB,0} = D_{CB,0} (55)$$

<sup>&</sup>lt;sup>27</sup>Theoretically, we could also look at a policy where the home central bank directly acquires foreign investment securities instead of foreign bonds. However, this seems little realistic as the foreign investment market is rather the responsibility of the foreign policy makers/central bank.

<sup>&</sup>lt;sup>28</sup>When no agency friction applies to the intermediation of reserves, the banks will not be constrained in their funding of these assets and the central bank is able to elastically issue domestic bonds (in the form of reserves) to fund its asset purchases.

Its profit in period 1, which will be transferred to the home households, is given by:

$$\Pi_{CB,1} = (R_{k,1} - R_1)p_0 S_{CB,0} + \left(R_1^* \frac{e_1}{e_0} - R_1\right) e_0 A_{CB,0}$$
(56)

The consolidation of the domestic households', the banks' and the central bank's budget constraints shows how the central bank's purchases of foreign assets alter the market clearing condition on the international credit market:

$$e_0 A_{p,0} + e_0 A_{CB,0} + e_0 f - f^* = p_0 N X_0$$
(57)

The excess supply of foreign bonds, resulting from the home country's trade surplus as well as the home and foreign households' portfolio flows, are now jointly absorbed by the home banks and central bank. Gross capital outflows  $(e_0A_{p,0} + e_0A_{CB,0} + e_0f)$  now also comprise the home central bank's purchases of foreign bonds.

In the investment market, market clearing condition (35) changes to

$$K_1 = S_{p,0} + S_{CB,0} (58)$$

and reflects the fact that also capital in the home country is now partly intermediated publicly. Finally, market clearing condition (38) becomes:

$$B_0 = D_{p,0} + D_{CB,0}. (59)$$

As the home central bank can just intervene in the domestic investment market or in the international credit market, only the home banks' incentive constraint is relevant for determining whether its interventions have an effect. When the home banks' balance sheet constraint is not binding, returns on the international credit and the home investment market are determined by frictionless arbitrage and interventions by the central bank are neutral. Its purchases of either home securities or foreign assets simply replace part of private intermediation, but have no effect on prices and the exchange rate. However, central bank interventions are non-neutral in markets where the financial friction parameters are high enough to generate limits to arbitrage. When banks are balance sheet constrained on one or both of the two markets, central bank purchases of the respective assets do not just replace private intermediation one by one, but rather expand the total demand for the respective asset type, which in turn drives down the excess return(s). The precise effect on different excess returns depends on whether the balance sheet constraint is binding due to positive friction parameters in just one or both of the two markets. In the former case, central bank interventions have only an effect on excess returns in the market that exhibits limits to arbitrage while the other market is unaffected. In the latter case, purchases of either asset affect excess returns in both markets. This spillover effect results from no-arbitrage relation (28). For better intuition, consider the example of the central bank intervening by purchasing foreign bonds.

According to the above reasoning, this reduces excess returns in the international credit market. The home banks thereupon adjust their portfolio towards investment securities, which have due to the central bank intervention a relatively higher excess return. This in turn pushes down excess returns in the investment market as well, until the excess returns in the international credit and the investment market, adjusted by the weight  $\frac{\theta}{\Delta}$ , are equalized. As already noted earlier, equation (28) also implies that excess returns are higher in the market where the banks face higher limits to arbitrage. Thus, everything else equal, central bank interventions will make excess returns move by more (in absolute terms) in the market where banks are most constrained. And importantly, this holds no matter which of the two assets the central bank purchases. In the limit, intermediation by the home central bank can make the excess returns on the home investment market and the international credit market disappear completely. As long as  $\theta^*$  is small enough for the foreign banks' capital constraints not to be binding, the economy will then be back in the frictionless state. But remember that, realistically, one would have to introduce efficiency costs to central bank intermediation (as mentioned above), which in turn could make such extreme interventions less desirable.

The additional intermediation by the central bank allows for higher levels of home capital and net exports to be intermediated, as can be observed in the new capital constraint:

$$CC = \begin{cases} \frac{\Delta}{\Delta - \frac{1}{R_{1}} \left(R_{1}^{*} \frac{e_{1}}{e_{0}} - R_{1}\right)} p_{0} N_{0} \geq \theta p_{0} (K_{1} - S_{CB,0}) + \Delta (p_{0} N X_{0} - e_{0} f + f^{*} - e_{0} A_{CB,0}) \\ & \text{if } \theta \geq 0, \Delta > 0 \\ \frac{\theta}{\theta - \frac{1}{R_{1}} (R_{k,1} - R_{1})} p_{0} N_{0} \geq \theta p_{0} (K_{1} - S_{CB,0}) + \Delta (p_{0} N X_{0} - e_{0} f + f^{*} - e_{0} A_{CB,0}) \\ & \text{if } \theta > 0, \Delta \geq 0 \\ \text{no CC} & \text{if } \theta = 0, \Delta = 0 \end{cases}$$

$$(60)$$

The inequalities in (60) will hold with equality given that we are considering here the case where banks are balance sheet constrained. They show how central bank asset purchases  $p_0S_{CB,0}$  and  $e_0A_{CB,0}$ , respectively, reduce the amount of funds that need to be intermediated by the home banks, so as to relax their capital constraint and bring intermediated quantities closer to their frictionless level.

Furthermore, equation (60) reveals that in terms of the amount of intervention needed in order to reach a given reduction of excess returns, it matters which asset the central bank buys, even when banks are equally constrained on the home investment market and the international credit market and spillover effects reduce excess returns in the two of them. If both markets are affected by limits to arbitrage, buying a certain amount of investment securities  $p_0S_{CB,0}$  relaxes the constraint to the same extent, and therefore has exactly the same effect, as buying foreign assets  $e_0A_{CB,0}$  in the amount of  $\frac{\theta}{\Delta}p_0S_{CB,0}$ . Intuitively, a central bank intervention involving the issuance of a certain amount of domestic bonds frees up a higher amount of bank capital in the market that faces higher limits to arbitrage. This implies that when  $\Delta > \theta$ , i.e. when the international credit market

exhibits higher excess returns than the home investment market, foreign exchange interventions have a stronger effect than credit easing and should be preferred to the latter in order to avoid an unnecessary expansion of the central bank's balance sheet. Likewise, credit easing would be the preferred instrument when  $\theta > \Delta$ .

An overview of the model's equilibrium equations with portfolio flows and (home) central bank intervention is provided in Appendix C. Figures 9 and 10 provide numerical illustrations of the effects of credit easing and foreign exchange interventions, respectively, when banks are equally constrained on both markets ( $\theta = \Delta$ ) and  $\theta^* = 0$ . As described before, both, credit easing and foreign exchange interventions, can positively affect excess returns and intermediated levels of funds. However, to make excess returns on these two markets disappear completely and bring the economy back to the frictionless equilibrium, it may not be enough to intervene in just one single market. It may be the case that the amount of assets outstanding in one market alone is too small such that the economy is still constrained even once the central bank has bought all of it. As an example, see Figure 10, where even when the central bank has absorbed all the excess supply of foreign bonds from the home banks ( $e_0A_{CB,0} = p_0NX_0$  and  $e_0A_{p,0} = 0$ ), the home banks are still credit constrained ( $\lambda > 0$ ). Numerical illustrations for when  $\Delta = \theta$  and  $\theta^* > 0$  are provided in Figures A.7 and A.8 in Appendix C.

Obviously, similar to the home central bank, also the foreign central bank can release potential tension on the foreign investment market and reduce the respective excess returns. As the underlying mechanisms essentially correspond to the ones just discussed for interventions by the home central bank, we refrain from going into more details.

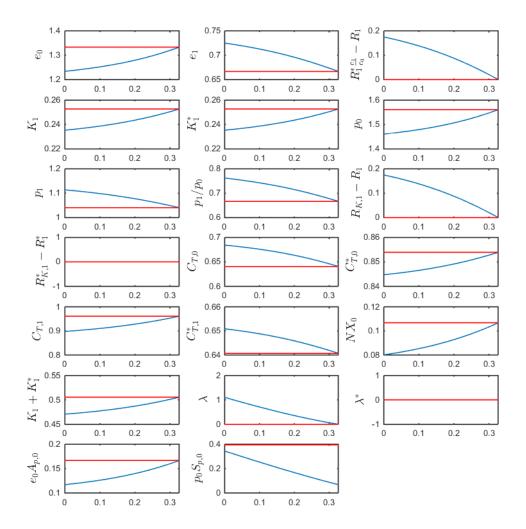


Figure 9: Effect of an increase in  $S_{CB,0}$  on different variables ( $p_0S_{CB,0}$  on x-axis). Evolution of the model's equilibrium as central bank intermediation in the domestic investment market increases starting from a point where there are limits to arbitrage in the domestic investment market and the international credit market.  $\theta = 1/3$ ,  $\theta^* = 0$ ,  $\Delta = 1/3$ , the remaining parameter values are summarized in Table A.1. The red line represents the value in the frictionless equilibrium.

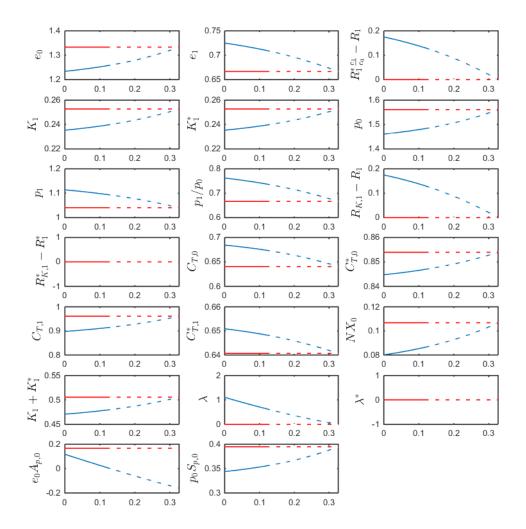


Figure 10: Effect of an increase in  $A_{CB,0}$  on different variables ( $e_0A_{CB,0}$  on x-axis). Evolution of the model's equilibrium as central bank intermediation in the international credit market increases starting from a point where there are limits to arbitrage in the domestic investment market and the international credit market.  $\theta = 1/3$ ,  $\theta^* = 0$ ,  $\Delta = 1/3$ , the remaining parameter values are summarized in Table A.1. The red line represents the value in the frictionless equilibrium. The dashed part of the lines captures the range of foreign exchange interventions where the latter would require the home banks to go short in foreign bonds in order to fulfill the central bank's demand for these assets and hence covers a part where our model technically is not valid.

# 7 Policy Response to Appreciation Pressure

Even tough simple, our model is still meant to capture the fundamental structure of the domestic and international bond markets and thereby allows to draw some interesting conclusions on the effectiveness of the unconventional monetary policy tools credit easing and foreign exchange (FX) interventions under various circumstances. In this last section, we want to shed light on three sources of external appreciation pressure related to credit market frictions, and discuss the respective policy options that a central bank has to reverse the effects on the current exchange rate and to mitigate the negative consequences on international trade. Depending on the source of appreciation pressure and the prevailing frictions in the domestic and international financial market, FX interventions and/or credit easing (or neither of them) are possible instruments.

The three shocks that we consider are financial frictions in the international credit market ( $\Delta \uparrow$ ), portfolio inflows ( $f^* \uparrow$ ), and financial frictions in the foreign investment market ( $\theta \uparrow$ ). In Figures 11(a) and 11(b), we provide an overview of how these shocks and the two policies affect the domestic banks' capital constraint, which lies at the core of our model, for different values of the friction parameters of the international credit and the foreign investment market. More precisely, for a given level of  $\theta$  (where by assumption  $\theta > 0$ ), these figures show where in the  $\theta^*\Delta$ -plane the home capital constraint is binding (shaded area), and how the shocks and central bank intermediation affect its general tightness. The line at the border of the shaded area corresponds to the critical values of  $\Delta$  at which the capital constraint starts to be binding and hence portfolio flows and central bank intervention become effective. For low levels of  $\theta^*$ , the foreign banks' capital constraint is not binding and, hence, the critical value of  $\Delta$  does not depend on the level of  $\theta^*$ . Once the foreign banks are constrained, the home banks need to absorb less excess supply of foreign bonds which makes them more risk-resistant such that they can face higher levels of  $\Delta$  before being restricted. In the area above this curve the home capital constraint binds tighter and this tightness increases the more the economy moves to the upper left.

The first source of external appreciation pressure that we focus on is financial frictions in the international credit market as they can arise for instance when markets start to distrust a certain foreign currency. An example for why such distrust might arise is the European sovereign debt crisis, which resulted in a depreciation of the euro against other major currencies. Within the framework of our model, this scenario can be captured by a (further) increase in  $\Delta$ . As described in sections 4.2 and 4.3, this leads, on the one hand, to a change in future relative output due to a change in the international allocation of capital and herewith to a home appreciation in both periods. On the other hand, it results in an increase in the interest rate spread, i.e. a lower foreign depreciation between today and the future, reflected in additional home appreciation pressure in the current period. A country facing appreciation pressure from this type of market imperfection has at least one possible policy tool it can rely on. As described in section 6, FX interventions, i.e. purchases of foreign bonds by the central bank,  $A_{CB,0}$ , reduce the excess supply of such bonds which needs to be absorbed by private intermediaries. The home banks' capital constraint is relaxed, which translates into an upward shift in the "critical- $\Delta$ -curve in Figure 11(b)). This, in turn, results in a reduction of the interest rate spread and a home depreciation in the current period. Suppose now that apart from the international credit market, the home investment market exhibits limits to arbitrage as well. Such a situation may arise when financial frictions reach a global extent as was observable for instance during the recent financial crisis, which raised fears of the potential

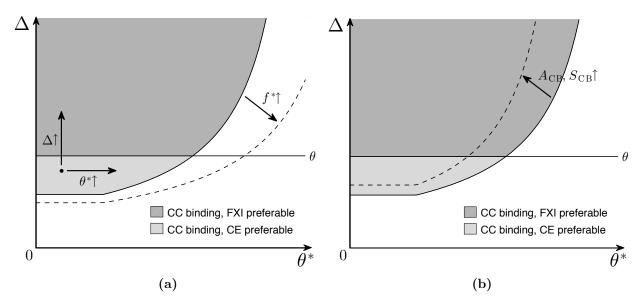


Figure 11: Overview of possible constellations of parameters  $\Delta$  and  $\theta^*$  for a given  $\theta$ . The home banks' capital constraint (CC) is binding in the shaded area only. International portfolio flows will affect the general tightness of the CC (see Figure (a)) and will have an impact on excess returns whenever the CC is binding. Likewise, central bank interventions can be used to generally relax the CC (see Figure (b)). Whenever the CC is binding, such interventions will be effective in lowering excess returns. FX interventions should be given priority over credit easing whenever  $\Delta > \theta$ .

collapse of large financial institutions all over the world. In such a situation, the home central bank has an additional policy tool at its disposal. As also argued in section 6, credit easing, i.e. central bank purchases of investment securities,  $S_{CB,0}$ , and FX interventions are close substitutes whenever financial frictions start to matter in both the home investment market as well as the international credit market as it is the case during a global financial crisis. Both policy tools relax the banks' capital constraint and the portfolio rebalancing of private intermediaries makes sure that the relative excess returns of the two markets remain constant. Hence, like FX interventions, credit easing leads to an upward shift of the "critical- $\Delta$ "-curve in Figure 11. In order to avoid an unnecessary large expansion of its balance sheet, the central bank should intervene in the market with higher frictions. Thus, should for some reason the limits to arbitrage in the home investment market be larger than the limits to arbitrage in the international credit market, then credit easing is more desirable for stemming appreciation pressure than direct interventions in the international credit market. Note, however, that whichever type of intervention the central bank chooses, it will not be able to bring the economy back to its initial state, i.e. the state before the shock to the international credit market, unless this initial state was a frictionless one. The increase in  $\Delta$  has permanently altered no-arbitrage relation (28) and hence the, from the banks' perspective, optimal relative size of the excess returns in the investment and international credit market.

A second external source of appreciation pressure is an increase in portfolio inflows  $f^*$ , as experienced by countries like Switzerland, Denmark or Israel during the global financial and European

sovereign debt crisis.<sup>29</sup> As discussed in section 5, this involves an increase in the excess supply of foreign bonds that the home banks need to absorb in equilibrium and therefore results in a widening of the spread in the interest rate parity and hence a home appreciation in period 0 if banks are constrained on the international credit market, i.e. if  $\Delta$  is high enough for the home banks' incentive constraint being or starting to be binding. Figure 11(a) shows how portfolio inflows lead to a downward shift in the "critical- $\Delta$ "-curve, reflecting that for given levels of the parameters  $\theta, \theta^*$  and  $\Delta$ , the banks will be subject to a higher general restrictiveness of the CC. The policy options of the home central bank when facing this type of appreciation pressure are the same as in the case of an increase in  $\Delta$ . Whenever capital flows have an impact on the exchange rate, FX interventions, which in the end just are a special type of capital flows, will as well. By choosing  $e_0A_{CB,0}=f^*$  (where  $e_0$  is equal to the pre-shock value), the central bank's purchases of foreign bonds can theoretically fully reverse the increase in the excess supply of foreign bonds and herewith prevent the appreciation. By absorbing all portfolio inflows, the central bank can shift the "critical- $\Delta$ "-curve in Figure 11 back to its original position and thereby offset the effect on the exchange rate. A prominent example of a central bank addressing capital inflows by FX interventions was the Swiss National Bank between March 2009 and June 2010, where it purchased considerable amounts of foreign bonds to stem the upward pressure on the Swiss franc and to prevent a tightening of monetary conditions.

Again, credit easing can interestingly achieve exactly the same goal as FX interventions if banks are constrained on the market for home investment securities as well  $(\theta > 0)$ . The central bank's acquisitions of home investment securities can likewise free up risk bearing capacity of the home banks, which these in turn can use for absorbing the increased excess supply of foreign bonds and reducing the spread in the interest rate parity. As we can see from equation (60), purchases of home investment securities in the amount of  $p_0S_{CB,0} = \frac{\Delta}{\theta}f^*$  (where  $p_0$  is equal to the pre-shock value) even allows to bring the economy back to the state it was in prior to the portfolio inflow shock. Because both policies' mechanisms are in the end the same, they are both able to shift the "critical- $\Delta$ "-curve in Figure 11 back to its original position and thereby offset the effect on the spreads. Once again, it is more costly for the central bank to intervene in the market with lower limits to arbitrage, i.e. credit easing should be the preferred policy whenever  $\theta > \Delta$ . However, there is a major limit to credit easing as it cannot exceed the level of home capital  $p_0K_1$ . Once all this capital is owned by the central bank, credit easing is no more feasible. As an example, consider again the case of Switzerland. In 2009, besides the interventions on the international credit market, the Swiss National Bank also started carrying out a bond purchase program. The goal was to relax the conditions on capital markets and thereby improving monetary transmission. Compared to the programs of other countries the amount of bonds purchased relative to GDP was rather small. The program was stopped again in 2010. As it has turned out by now, even buying out the whole Swiss bond market would not have been sufficient to relax the upward pressure on the Swiss franc. While

<sup>&</sup>lt;sup>29</sup>Obviously, a decrease in portfolio outflows  $e_0 f$  would have the same effects as an increase in portfolio inflows  $f^*$ , but would not be classified as an external shock.

the total capitalization of the Swiss bond market in 2009 was around CHF 530 billion<sup>30</sup>, the Swiss National Bank's foreign currency investments have increased by CHF 580 billion since 2009.

The third and last external source of appreciation pressure related to credit market frictions in our model is a financial crisis in the foreign country, coming in the form of frictions in the foreign investment market which is captured by an increase in  $\theta^*$ . Note that in the model the foreign country can also be interpreted as the rest of the world. During the recent financial crisis, countries like Australia, Canada, China, Israel, Korea, Norway and Singapore did not suffer from a financial crisis themselves, but were negatively affected by the global consequences of the turmoil on US and European markets, i.e. by financial frictions abroad. As described in sections 4.1 and 4.3, this induces an international reallocation of capital away from the foreign country towards the home country, leading to a relative increase in the home country's lifetime resources and hence to a permanent home appreciation, i.e. an appreciation in both periods. If banks are not constrained on the international credit market, this is the only effect on the exchange rate. However, if banks are also constrained on the international credit and/or the home investment market, an increase in  $\theta^*$  and the resulting drop in net outflows may bring the economy to a state where the home banks' CC is relaxed (see Figure 11(a)) and therefore result in a decrease in excess returns in the international credit and/or the home investment market. Hence, as compared to the cases in the two previous paragraphs, this type of appreciation pressure is solely based on a change in fundamentals. In this respect, it is "justified", as opposed to any (temporary) appreciation pressure coming from an increase in the spread in the interest rate parity or safety premium on domestic bonds, respectively, as is experienced when there are limits to arbitrage in the international credit market or an increase in capital inflows. The reason why more funds are invested domestically is simply that the financial crisis abroad leads to relatively better investment opportunities at home and hence, to less capital outflows. It follows that when there is appreciation pressure due to frictions in the foreign investment market, the home central bank has no reason to intervene in the first place. In addition, when the home banks are not constrained on the international credit market ( $\Delta = 0$  or  $\Delta$  and  $\theta$  small enough not to be binding), it anyway has no possibility to affect the exchange rate, be it through FX interventions or credit easing. When there are limits to arbitrage on the international credit market, it would have this possibility, but purchases of foreign bonds (or home investment securities, if  $\theta > 0$ ) merely lead to a decrease in the safety premium on domestic bonds: While they do release appreciation pressure in the first period, they put further upward pressure in the second period. Such purchases only address the capital misallocation and exchange rate distortion caused by the international credit market frictions, but not the appreciation pressure caused by the frictions in the foreign investment market. Accordingly, the home central bank has no incentive to take action when its country faces this type of appreciation pressure.

Table 2 provides an overview of the sources of appreciation pressure and the possible policy response(s). Obviously, in practice one would find countries facing several of the mentioned shocks at the same time.

<sup>&</sup>lt;sup>30</sup>Source: SIX

	Source of Appreciation Pressure		
Market(s) with Limits to Arbitrage (after Shock)	$\Delta\uparrow$	$f^* \uparrow$	$ heta^*\uparrow$
Internat. credit only Home inv. only Internat. credit & home inv.	$FXI \\ n/a^c \\ FXI/CE^a$	$\begin{array}{c} {\rm FXI} \\ {\rm n/a}^c \\ {\rm FXI/CE} \end{array}$	$FXI^{ab}$ none $FXI/CE^{ab}$

<sup>&</sup>lt;sup>a</sup>The central bank cannot reverse the effect of the shock.

<sup>b</sup>There is a trade-off between curbing home appreciation today and tomorrow.

**Table 2:** Overview of sources of appreciation pressure and the possible policy response(s).

Finally, note that our model is a real framework and therefore in this form not suitable for analysing the effects of extensive expansionary monetary policies like quantitative easing because expansionary monetary policy always involves an increase in money supply. Consider the following example. At first sight, the effects of the ECB's quantitative easing program seem to be at odd with the predictions of our model which suggest that a central bank's asset purchases lead to a shift in capital towards the respective country, which will then experience an appreciation. The ECB's quantitative easing program, on the other side, triggered a devaluation of the euro. However, once the huge increase in money supply this involves is taken into account, this is not surprising anymore. By extending our model and introducing, for instance, a simple money-in-the-utility utility function, our framework would be able to reflect nominal effects and would also predict a devaluation. While the higher demand for assets as before would lead to a decrease in spreads, the change in relative money supply would now lead to depreciation pressure.

### 8 Conclusion

We provide a simple two-country framework with financial frictions that allows to think about and compare foreign exchange interventions and credit easing. These two policies are effective through the same channel, namely the portfolio balance channel. Limited commitment of financial intermediaries and the resulting credit constraint lead to positive excess returns in the domestic and foreign investment markets as well as the international credit market. When banks are financially constrained and only then, international portfolio flows and central bank intermediation have real effects as they alter the excess supply of assets which needs to be absorbed by the banks. Within this framework, we have looked at three external sources of appreciation pressure related to financial frictions and have identified the respective policy options that a central bank has. An increase in the frictions in the international credit market and capital inflow shocks both result in an increase in the safety premium on domestic bonds and hence a temporary home appreciation. In these two

<sup>&</sup>lt;sup>c</sup>With this source of appreciation pressure, the international credit market exhibits limits to arbitrage by assumption.

cases, foreign exchange intervention can reverse the appreciation pressure. An increase in the limits to arbitrage in the foreign investment market triggers an appreciation as well, but in this case, the appreciation is permanent and purchases of foreign bonds cannot reverse it. We have furthermore argued that if banks are constrained on the home investment market as well, credit easing is a substitute to foreign exchange interventions and can achieve the same goal as there are spillover effects across the different markets. The effectiveness of the two different types of policies, however, might differ. Intervention will come at lower costs if it targets the market that faces the highest excess returns.

Concerning such policy implications one has to keep in mind that it is usually difficult to identify the exact source of credit spreads and appreciation pressure. The causes discussed in this paper, i.e. frictions in financial markets and capital inflow shocks, might be difficult to distinguish from other reasons like changes in expectations about future monetary policy and nominal frictions, to name only some of them. On this account, it might be hard to assess whether there are frictions at all and hence whether foreign exchange interventions and credit easing would be effective.

The model supports the macro-prudential reforms of the Basel Committee on Banking Supervision (Basel III) in the sense that higher minimum capital requirements improve the stability of the banking sector. Higher levels of bank equity represent a higher amount of risk-absorbing capital which in turn relaxes the balance sheet constraint of banks. Hence, if banks are required to raise a large part of their funds trough equity financing, the likelihood and severity of financial frictions can be lowered.

Several extensions could be interesting for future research. The introduction of money and eventually nominal frictions would be a straightforward extension of our model, as the example on the ECB's quantitative easing program at the end of Section 7 showed. Regarding domestic asset purchase programs, our model allows to analyze purchases of securities with some private risks, while the acquisition of long-term government bonds is not implemented. This, and the introduction of uncertainty are other possible extensions of the model. Moreover, the two policy tools discussed in this paper potentially entail a large expansion of the central bank's balance sheet. However, our model does not address the risks associated with such a balance sheet expansion. This is a topic that concerns central banks like the Federal Reserve, the Bank of England, the ECB or the Swiss National Bank that have conducted sizeable asset purchase programs which have resulted in a dramatic expansion of their balance sheet. Research in this direction is left for future work.

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# A Equilibrium Equations and Numerical Solution of the Model

## A.1 Equilibrium Equations

Our model contains the following 38 endogenous variables:  $C_{NT,0}$ ,  $C_{NT,1}$ ,  $C_{T,0}$ ,  $C_{T,1}$ ,  $p_0$ ,  $p_1$ ,  $w_1$ ,  $Y_{T,1}$ ,  $R_1$ ,  $R_k$ ,  $R_k$ ,  $R_1$ ,  $R_1$ ,  $R_1$ ,  $R_2$ ,  $R_3$ ,  $R_4$ ,  $R_5$ ,  $R_5$ ,  $R_5$ ,  $R_6$ 

Since we set  $Y_{NT,0} = Y_{NT,1} = \chi$  it follows from the combination of equations (4), (10), (43) and (44) that the domestic households' Euler condition reduces to  $R_1 = \frac{1}{\beta}$ . Equivalently, we have  $R_1^* = \frac{1}{\beta^*}$ . The combination of equations (2) and (3) yields the intertemporal budget constraint of the domestic household

$$C_{NT,1} + p_1 C_{T,1} = R_1 (p_0 Y_{T,0} - p_0 N_0 + Y_{NT,0} - C_{NT,0} - p_0 C_{T,0}) + w_1 L + p_1 N_1 + Y_{NT,1}$$

Using the market clearing condition for nontraded goods, the market clearing condition for labor and equation (17), the intertemporal budget constraint simplifies to

$$\underbrace{1}_{p_1C_{T,1}} = R_1 \underbrace{\left(p_0 Y_{T,0} - p_0 N_0 - \underbrace{1}_{p_0 C_{T,0}}\right)}_{B_0} + \underbrace{\left(1 - \alpha\right) \left(\frac{K_1}{L}\right)^{\alpha} p_1 L + p_1 N_1}_{w_1} \tag{61}$$

Consumption expenditure in period 1 depends on the savings in domestic bonds in period 0, on the wage in period 1 and on the profit of the bank the household owns. Note that consumption expenditure on the traded good is constant and equal to 1.

Using equation (16) and the market clearing condition for labor we can rewrite the return on home securities as follows

$$R_{k,1} = \left(\alpha \left(\frac{L}{K_1}\right)^{1-\alpha} + (1-\delta)\right) \frac{p_1}{p_0} \tag{62}$$

Equivalently, we can simplify the return on foreign securities

$$R_{k,1}^* = \left(\alpha \left(\frac{L^*}{K_1^*}\right)^{1-\alpha} + (1-\delta)\right) \frac{p_1 e_0}{p_0 e_1} \tag{63}$$

The value of the home bank's equity capital in period 1 is (from equation (25))

$$p_1 N_1 = \left(R_{k,1} - R_1\right) p_0 S_{p,0} + \left(R_1^* \frac{e_1}{e_0} - R_1\right) e_0 A_{p,0} + R_1 p_0 N_0 \tag{64}$$

Using the market clearing condition for claims on domestic firms and equation (47), the value of

the home bank's equity capital in period can be rewritten as

$$p_1 N_1 = (R_{k,1} - R_1) p_0 K_1 + \left( R_1^* \frac{e_1}{e_0} - R_1 \right) \underbrace{(p_0 Y_{T,0} - p_0 K_1 - 1)}_{p_0 N X_0} + R_1 p_0 N_0$$
 (65)

The first order conditions of the domestic bank, equations (26) and (27), can be slightly simplified to

$$\frac{1}{R_1}\left(R_{k,1} - R_1\right) = \frac{\lambda}{1+\lambda}\theta\tag{66}$$

$$\frac{1}{R_1} \left( R_1^* \frac{e_1}{e_0} - R_1 \right) = \frac{\lambda}{1 + \lambda} \Delta \tag{67}$$

And the first order condition of the foreign bank, equation (33), is now

$$\frac{1}{R_1^*} \left( R_{k,1}^* - R_1^* \right) = \frac{\lambda^*}{1 + \lambda^*} \theta^* \tag{68}$$

Under the assumption that  $\theta > 0$  and  $\Delta > 0$  we can rewrite the domestic incentive constraint (23) as a capital constraint (CC) (use equations (25) - (28))

$$V_{0} \geq \theta p_{0} S_{p,0} + \Delta e_{0} A_{p,0}$$

$$\Leftrightarrow \Lambda_{0,1} \left( (R_{k,1} - R_{1}) p_{0} S_{p,0} + \left( R_{1}^{*} \frac{e_{1}}{e_{0}} - R_{1} \right) e_{0} A_{p,0} + R_{1} p_{0} N_{0} \right) \geq \theta p_{0} S_{p,0} + \Delta e_{0} A_{p,0}$$

$$\Leftrightarrow \Lambda_{0,1} \left( R_{1}^{*} \frac{e_{1}}{e_{0}} - R_{1} \right) \left( \frac{\theta}{\Delta} p_{0} S_{p,0} + e_{0} A_{p,0} \right) + \Lambda_{0,1} R_{1} p_{0} N_{0} \geq \Delta \left( \frac{\theta}{\Delta} p_{0} S_{p,0} + e_{0} A_{p,0} \right)$$

$$\Leftrightarrow \Lambda_{0,1} R_{1} p_{0} N_{0} \geq \left( \frac{\theta}{\Delta} p_{0} S_{p,0} + e_{0} A_{p,0} \right) \left( \Delta - \Lambda_{0,1} \left( R_{1}^{*} \frac{e_{1}}{e_{0}} - R_{1} \right) \right)$$

$$\Leftrightarrow \frac{\Lambda_{0,1} R_{1} p_{0} N_{0}}{\Delta - \Lambda_{0,1} \left( R_{1}^{*} \frac{e_{1}}{e_{0}} - R_{1} \right)} \geq \frac{\theta}{\Delta} p_{0} S_{p,0} + e_{0} A_{p,0}$$

$$\Leftrightarrow \phi p_{0} N_{0} \geq \theta p_{0} S_{p,0} + \Delta e_{0} A_{p,0}$$

$$\Leftrightarrow \phi p_{0} N_{0} \geq \theta p_{0} S_{p,0} + \Delta e_{0} A_{p,0}$$

$$(69)$$

where  $\phi = \frac{\Delta\Lambda_{0,1}R_1}{\Delta-\Lambda_{0,1}\left(R_1^*\frac{e_1}{e_0}-R_1\right)} = \frac{\theta\Lambda_{0,1}R_1}{\theta-\Lambda_{0,1}\left(R_{k,1}-R_1\right)}$ . If in one of the two markets the friction parameter is set to zero, then this inequality simplifies to

$$\frac{\Lambda_{0,1}R_1}{\Delta - \Lambda_{0,1} \left( R_1^* \frac{e_1}{e_0} - R_1 \right)} p_0 N_0 \ge e_0 A_{p,0} \quad \text{if} \quad \theta = 0$$
 (70)

$$\frac{\dot{\Lambda}_{0,1}R_1}{\theta - \Lambda_{0,1} (R_{k,1} - R_1)} p_0 N_0 \ge p_0 S_{p,0} \quad \text{if} \quad \Delta = 0$$
 (71)

Note that if  $\theta > 0$  and/or  $\Delta > 0$  this does not necessarily imply that the capital constraint is binding  $(\lambda > 0)$ , i.e. that there are limits to arbitrage in at least one market. The capital constraint is only

binding if  $\theta > \bar{\theta}$  and/or  $\Delta > \bar{\Delta}$ . We can summarize the capital constraint as follows

$$CC = \begin{cases} \frac{\Delta \Lambda_{0,1} R_1}{\Delta - \Lambda_{0,1} \left( R_1^* \frac{e_1}{e_0} - R_1 \right)} p_0 N_0 \ge \theta p_0 S_{p,0} + \Delta e_0 A_{p,0} & \text{if} & \theta \ge 0, \Delta > 0 \\ \frac{\theta \Lambda_{0,1} R_1}{\theta - \Lambda_{0,1} \left( R_{k,1} - R_1 \right)} p_0 N_0 \ge \theta p_0 S_{p,0} + \Delta e_0 A_{p,0} & \text{if} & \theta > 0, \Delta \ge 0 \\ \text{no CC} & \text{if} & \theta = 0, \Delta = 0 \end{cases}$$
(72)

If  $\theta > 0$  and  $\Delta > 0$  it is irrelevant whether the first or the second equation of (72) is considered.

Using the Euler condition, the market clearing condition for claims on domestic firms and equation (47) we can simplify the capital constraint of the domestic bank as follows

$$CC = \begin{cases} \frac{\Delta}{\Delta - \frac{1}{R_1} \left( R_1^* \frac{e_1}{e_0} - R_1 \right)} p_0 N_0 \ge \theta p_0 K_1 + \Delta \left( p_0 Y_{T,0} - p_0 K_1 - 1 \right) & \text{if} & \theta \ge 0, \Delta > 0 \\ \frac{\theta}{\theta - \frac{1}{R_1} \left( R_{k,1} - R_1 \right)} p_0 N_0 \ge \theta p_0 K_1 + \Delta \left( p_0 Y_{T,0} - p_0 K_1 - 1 \right) & \text{if} & \theta > 0, \Delta \ge 0 \\ \text{no CC} & \text{if} & \theta = 0, \Delta = 0 \end{cases}$$

Keep in mind that  $(p_0Y_{T,0} - p_0K_1 - 1) = p_0NX_0$ . For any parameter specification but  $\theta = \Delta = 0$  the Karush-Kuhn-Tucker (KKT) conditions need to hold. Define

$$g \equiv \begin{cases} \frac{\Delta}{\Delta - \frac{1}{R_1} \left( R_1^* \frac{e_1}{e_0} - R_1 \right)} p_0 N_0 - \theta p_0 K_1 - \Delta (p_0 Y_{T,0} - p_0 K_1 - 1) & \text{if } \theta \ge 0, \Delta > 0 \\ \frac{\theta}{\theta - \frac{1}{R_1} \left( R_{k,1} - R_1 \right)} p_0 N_0 - \theta p_0 K_1 - \Delta (p_0 Y_{T,0} - p_0 K_1 - 1) & \text{if } \theta > 0, \Delta \ge 0 \end{cases}$$
(73)

where g is a function of domestic endogenous variables. Then, the KKT conditions for the inequality constraint of the home bank are

$$g \ge 0 \tag{74}$$

$$\lambda \ge 0 \tag{75}$$

$$\lambda q = 0 \tag{76}$$

Equivalently, for the foreign bank we have

$$CC^* = \begin{cases} \frac{1}{\theta^* - \frac{1}{R_1^*} (R_{k,1}^* - R_1^*)} \frac{p_0}{e_0} N_0^* \ge \frac{p_0}{e_0} K_1^* & \text{if } \theta^* > 0\\ \text{no } CC^* & \text{if } \theta^* = 0 \end{cases}$$
 (77)

where we have used the law of one price. For any parameter specification but  $\theta^* = 0$  the KKT conditions need to hold. Define

$$g^* \equiv \left\{ \frac{1}{\theta^* - \frac{1}{R_1^*} (R_{k,1}^* - R_1^*)} \frac{p_0}{e_0} N_0^* - \frac{p_0}{e_0} K_1^* \quad \text{if} \quad \theta^* > 0 \right.$$

where  $g^*$  is a function of foreign endogenous variables. Then, the KKT conditions for the inequality

constraint of the foreign bank are

$$g^* \ge 0 \tag{78}$$

$$\lambda^* \ge 0 \tag{79}$$

$$\lambda^* g^* = 0 \tag{80}$$

The market clearing condition for traded goods in period 0 simplifies to

$$Y_{T,0} + Y_{T,0}^* = \frac{1}{p_0} + \frac{e_0}{p_0} + K_1 + K_1^*$$
(81)

The market clearing condition for traded goods in period 1 is

$$K_1^{\alpha} L^{1-\alpha} + K_1^{*\alpha} L^{*1-\alpha} + (1-\delta)K_1 + (1-\delta)K_1^* = \frac{1}{p_1} + \frac{e_1}{p_1}$$
(82)

where we have used the production function of the domestic and foreign firm.

In sum, we reduce our system of equations to one of the following 13 equations

$$R_1 = \frac{1}{\beta} \qquad (83)$$

$$R_1^* = \frac{1}{\beta^*} \qquad (84)$$

$$1 = R_1(p_0 Y_{T,0} - p_0 N_0 - 1) + (1 - \alpha) \left(\frac{K_1}{L}\right)^{\alpha} p_1 L + p_1 N_1$$
 (85)

$$R_{k,1} = \left(\alpha \left(\frac{L}{K_1}\right)^{1-\alpha} + (1-\delta)\right) \frac{p_1}{p_0} \qquad (86)$$

$$R_{k,1}^* = \left(\alpha \left(\frac{L^*}{K_1^*}\right)^{1-\alpha} + (1-\delta)\right) \frac{p_1 e_0}{p_0 e_1}$$
 (87)

$$p_1 N_1 = \left(R_{k,1} - R_1\right) p_0 K_1 + \left(R_1^* \frac{e_1}{e_0} - R_1\right) \left(p_0 Y_{T,0} - p_0 K_1 - 1\right) + R_1 p_0 N_0 \tag{88}$$

$$\frac{1}{R_1} (R_{k,1} - R_1) = \frac{\lambda}{1 + \lambda} \theta \qquad (89)$$

$$\frac{1}{R_1} \left( R_1^* \frac{e_1}{e_0} - R_1 \right) = \frac{\lambda}{1 + \lambda} \Delta \qquad (90)$$

$$\frac{1}{R_1^*} \left( R_{k,1}^* - R_1^* \right) = \frac{\lambda^*}{1 + \lambda^*} \theta^* \qquad (91)$$

$$CC = \begin{cases} \frac{\Delta}{\Delta - \frac{1}{R_1} \left( R_1^* \frac{e_1}{e_0} - R_1 \right)} p_0 N_0 \ge \theta p_0 K_1 + \Delta (p_0 Y_{T,0} - p_0 K_1 - 1) & \text{if} & \theta \ge 0, \Delta > 0 \\ \frac{\theta}{\theta - \frac{1}{R_1} \left( R_{k,1} - R_1 \right)} p_0 N_0 \ge \theta p_0 K_1 + \Delta (p_0 Y_{T,0} - p_0 K_1 - 1) & \text{if} & \theta > 0, \Delta \ge 0 \\ \text{no CC}, \lambda = 0 & \text{if} & \theta = 0, \Delta = 0 \end{cases}$$
(92)

$$CC^* = \begin{cases} \frac{1}{\theta^* - \frac{1}{R_1^*} (R_{k,1}^* - R_1^*)} \frac{p_0}{e_0} N_0^* \ge \frac{p_0}{e_0} K_1^* & \text{if } \theta^* > 0\\ \text{no } CC^*, \lambda^* = 0 & \text{if } \theta^* = 0 \end{cases}$$
(93)

$$Y_{T,0} + Y_{T,0}^* = \frac{1}{p_0} + \frac{e_0}{p_0} + K_1 + K_1^*$$
 (94)

$$K_1^{\alpha} L^{1-\alpha} + K_1^{*\alpha} L^{*1-\alpha} + (1-\delta)K_1 + (1-\delta)K_1^* = \frac{1}{p_1} + \frac{e_1}{p_1}$$
 (95)

Furthermore, we have to take into account the remaining KKT conditions for the domestic country if the domestic friction parameters are non-zero ( $\theta \neq 0$  and  $\Delta \neq 0$ ), equations (75) and (76), and the remaining KKT conditions for the foreign country if the foreign friction parameter is non-zero ( $\theta^* \neq 0$ ), equations (79) and (80). The 13 unknowns are  $e_0, e_1, p_0, p_1, K_1, K_1^*, R_1, R_1^*, R_{k,1}, R_{k,1}^*, N_1, \lambda, \lambda^*$ .

# A.2 Proof: Properties of the model when $\theta = \theta^* > 0$ and $\Delta = 0$ (for ICs binding)

First note that taking the ratio of the two expressions for the returns on the investment securities,  $R_{k,1}$  and  $R_{k,1}^*$  (see equations (86) and (87)), yields the following (generally valid) relationship

between  $K_1$  and  $K_1^*$  (for  $L = L^*$ ):

$$K_{1} = \left(\frac{\frac{p_{0}}{p_{1}} \frac{e_{1}}{e_{0}} R_{k,1}^{*} - (1 - \delta)}{\frac{p_{0}}{p_{1}} R_{k,1} - (1 - \delta)}\right)^{\frac{1}{1 - \alpha}} K_{1}^{*}$$
(96)

For finding the relative level of capital for when  $\theta = \theta^* > 0$  and  $\Delta = 0$  conditional on the ICs being binding, combine each country's (binding) capital constraints (see equations (92) and (93)) with the respective expression for the return on the investment securities:

$$\frac{1}{\theta - \frac{1}{R_1} \left( \left( \alpha \left( \frac{L}{K_1} \right)^{1-\alpha} + (1-\delta) \right) \frac{p_1}{p_0} - R_1 \right)} N_0 = K_1 \tag{97}$$

$$\frac{1}{\theta^* - \frac{1}{R_1^*} \left( \left( \alpha \left( \frac{L^*}{K_1^*} \right)^{1-\alpha} + (1-\delta) \right) \frac{p_1}{p_0} \frac{e_0}{e_1} - R_1^* \right)} N_0^* = K_1^*$$
(98)

By assumption,  $L^* = L$  and  $N_0^* = N_0$ . Furthermore,  $\Delta = 0$  implies that  $R_1 = R_{1e_0}^*$ , and we have  $\theta = \theta^* > 0$ . Thus, equation (98) can be written as:

$$\frac{1}{\theta - \frac{1}{R_1} \left( \left( \alpha \left( \frac{L}{K_1^*} \right)^{1 - \alpha} + (1 - \delta) \right) \frac{p_1}{p_0} - R_1 \right)} N_0 = K_1^*$$
(99)

Looking at equations (97) and (99), it becomes clear that it must be the case that  $K_1 = K_1^*$ . From  $K_1 = K_1^*$ , in turn, it follows that  $R_{k,1} = R_{k,1}^* \frac{e_1}{e_0}$  (see equation (96)), i.e. returns on the investment securities in terms of the home numéraire are equalized.

For evaluating the relative tightness of the two countries' banks' incentive constraints, note that the FOC's (26) and (33) can be written as (remember that  $\Lambda_{0,1} = \frac{1}{R_1}$  and  $\Lambda_{0,1}^* = \frac{1}{R_1^*}$ ):

$$\frac{R_{k,1}}{R_1} = 1 + \frac{\lambda}{1+\lambda}\theta\tag{100}$$

$$\frac{R_{k,1}^*}{R_1^*} = 1 + \frac{\lambda^*}{1 + \lambda^*} \theta^* \tag{101}$$

From  $R_1 = R_{1}^* \frac{e_1}{e_0}$  and  $R_{k,1} = R_{k,1}^* \frac{e_1}{e_0}$ , it follows that  $\frac{R_{k,1}}{R_1} = \frac{R_{k,1}^*}{R_1^*}$ . Hence, and given that  $\theta = \theta^*$ , equations (100) and (101) imply that  $\lambda = \lambda^*$ , i.e. the incentive constraints are equally binding in the two countries.

# A.3 Proof: Properties of the model when $\theta = \theta^* = \Delta > 0$ (for ICs binding)

 $\theta = \theta^* = \Delta > 0$  implies that  $R_1 < R_{k,1} = R_1^* \frac{e_1}{e_0} < R_{k,1}^* \frac{e_1}{e_0}$  (conditional on the ICs being binding). From  $R_{k,1} < R_{k,1}^* \frac{e_1}{e_0}$  and equation (96), it follows that  $K_1 > K_1^*$ .

For evaluating the relative tightness of the two countries' banks' incentive constraints, combine each country's (binding) capital constraints (see equations (92) and (93)) with FOCs (26) and (33)

and the respective market clearing conditions:

$$\frac{1}{\theta - \frac{\lambda}{1+\lambda}\theta} N_0 = K_1 + NX_0 \tag{102}$$

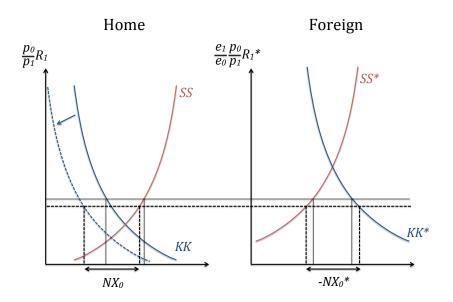
$$\frac{1}{\theta^* - \frac{\lambda^*}{1 + \lambda^*} \theta^*} N_0^* = K_1^* \tag{103}$$

Again, by assumption,  $N_0^* = N_0$ . Furthermore, we have  $\theta = \theta^*$ . Thus, taking the ratio of equations (102) and (103) yields:

$$\frac{1+\lambda}{1+\lambda^*} = \frac{K_1 + NX_0}{K_1^*} \tag{104}$$

We know that  $K_1 > K_1^*$  and  $NX_0 \ge 0$ , from which it finally follows that  $\lambda > \lambda^*$ , i.e. the home banks' incentive constraint binds tighter than the foreign banks' incentive constraint.

### A.4 Effect of Financial Frictions in the Home Investment Market



**Figure A.1:** Financial frictions in the home investment market:  $\theta > 0$ . The solid lines represent the frictionless equilibrium, the dashed lines the equilibrium with the friction.

This section discusses the effects of financial frictions in the home investment market, which are captured by an increase in the home investment market friction parameter  $\theta$ .  $\theta^*$  and  $\Delta$  are set to zero. When the home investment market friction parameter is sufficiently large for the home incentive constraint and hence also the endogenous capital constraint to become binding ( $\lambda > 0$ ), home banks are hindered to exploit all arbitrage opportunities and excess returns on the home investment market become positive:  $R_{k,1} - R_1 > 0$  (see equation (26)). Excess returns on the

international credit market and the foreign investment market, however, remain zero (see equations (27) and (33)). Combining the home banks' capital constraint (29) and the investment market clearing condition (35) reveals that with the constraint starting to be binding, the level of capital in the home country will obviously be limited:

$$\frac{1}{\theta - \frac{1}{R_1} (R_{k,1} - R_1)} p_0 N_0 \ge p_0 K_1 \tag{105}$$

(As  $\lambda > 0$ , this equation will hold with equality.)

Graphically, financial frictions in the home investment market shifts the home investment curve to the left (see Figure A.1)<sup>31</sup>. For a given real rate of return  $\frac{p_0}{p_1}R_1$ , investment in the home country decreases as the home banks' ability to intermediate funds in this market has decreased and it faces limits to arbitrage. Costs of capital in the home market increase. In order to maintain the world equilibrium, the equilibrium real rate of return has to decrease. Due to the frictions in the home investment market, the home country will in equilibrium on the one hand slightly decrease its savings, and on the other hand invest a much larger part abroad: The credit constraint with respect to investments in home capital makes the home banks reallocate their portfolio and invest a larger part in foreign bonds. Altogether, this causes an increase in the home country's net exports. The foreign country, on the other hand, also decreases its savings, but at the same time can increase its investments as the foreign banks obtain a larger amount of funds, which leads to an increase in its net imports. Overall, there is a decrease in world savings and, consequently, world investments, implying a lower level of world output in the second period. Furthermore, the frictions in the home investment market also lead to a misallocation of capital: Now, a majority of capital is invested in the foreign country. This change in the allocation of capital implies that relative to the frictionless level, the home country's output in the second period will decrease while the foreign country's output in the second period will increase, implying that there is a change in the two countries' fundamentals. The relative decrease in the home country's lifetime resources induces a home depreciation in both periods.

Figure A.2 provides a numerical illustration of these results. Setting  $\theta^*$  and  $\Delta$  equal to zero and using the calibration of Table A.1 for the remaining parameters, it shows how the model's equilibrium evolves as the home investment market friction parameter  $\theta$  increases. Note that  $\theta = 1$  (i.e. banks can divert *all* home investment securities) does not imply that home banks do not hold domestic assets anymore: It just means that in case of misbehaviour, the banks could divert and keep the proceeds of all these assets. If, however, the excess returns they can earn on the investment securities when not diverting them are large enough, they still have no incentive to misbehave and the financial markets will work even with  $\theta = 1$ .

 $<sup>^{31}</sup> For a formal proof of how an increase in <math display="inline">\theta$  affects the two countries' saving and investment schedules, see Appendix E.

## A.5 Numerical Illustration

As mentioned before, there exists no closed-form solution of the model, however, we can solve it numerically in Matlab. The following figures provide numerical results, in particular, they show the evolution of the model's equilibrium under different specifications of the friction parameters using the calibration of Table A.1 for the remaining parameters.

Table A.1: Parametrization

Domestic discount factor	β	1
Foreign discount factor	$eta^*$	0.5
Share of consumption expenditure spent on traded goods	$\frac{1}{1+\chi}$	0.5
Inelastic labor supply	$L,L^*$	1
Capital share	$\alpha$	0.33
Depreciation rate	$\delta$	0.33
Endowment of traded goods	$Y_{T,0}, Y_{T,0}^*$	1
Endowment of nontraded goods	$Y_{NT,0}, Y_{NT,1}, Y_{NT,0}^*, Y_{NT,1}^*$	χ
Bank's net worth	$N_0, N_0^*$	0.05

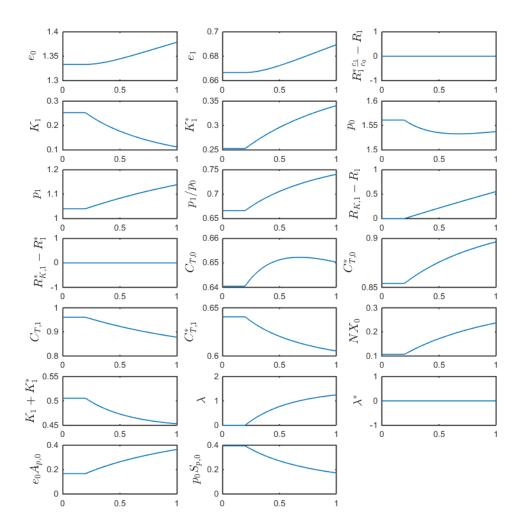


Figure A.2: Effect of an increase in  $\theta$  on different variables ( $\theta$  on x-axis). Evolution of the model's equilibrium as the friction parameter in the home investment market increases starting from a frictionless point.  $\theta \ge 0$ ,  $\theta^* = 0$ ,  $\Delta = 0$ , the remaining parameter values are summarized in Table A.1.

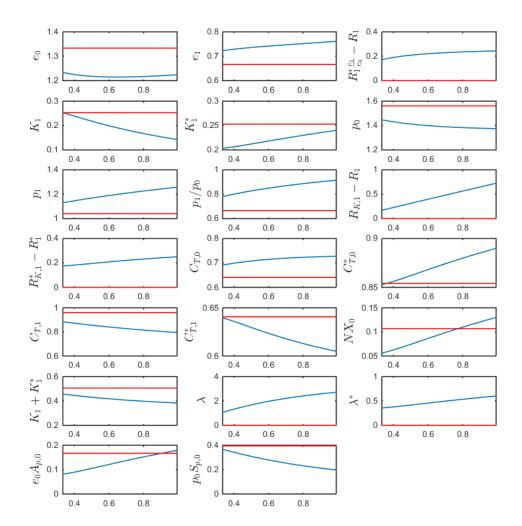


Figure A.3: Effect of an increase in  $\theta$  on different variables ( $\theta$  on x-axis). Evolution of the model's equilibrium as the friction parameter in the home investment market increases starting from a point where there are limits to arbitrage in all financial markets.  $\theta \geq 1/3$ ,  $\theta^* = 1/3$ ,  $\Delta = 1/3$ , the remaining parameter values are summarized in Table A.1. The red line represents the value in the frictionless equilibrium.

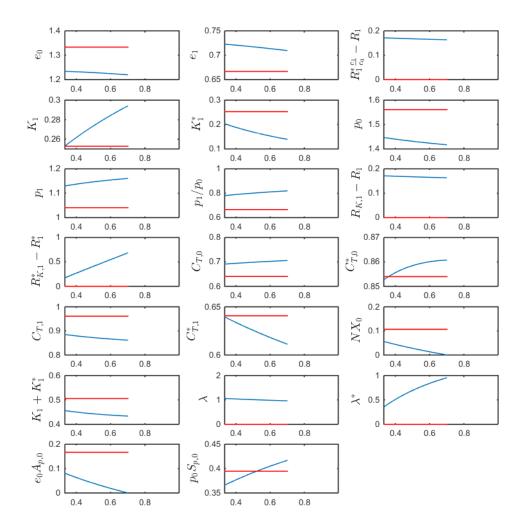


Figure A.4: Effect of an increase in  $\theta^*$  on different variables ( $\theta^*$  on x-axis). Evolution of the model's equilibrium as the friction parameter in the foreign investment market increases starting from a point where there are limits to arbitrage in all financial markets.  $\theta = 1/3$ ,  $\theta^* \ge 1/3$ ,  $\Delta = 1/3$ , the remaining parameter values are summarized in Table A.1. The red line represents the value in the frictionless equilibrium. Given that the set-up of the banking sector is only valid when  $e_0 A_{p,0} \ge 0$ , the plots only cover a limited range of possible values for  $\theta^*$ .

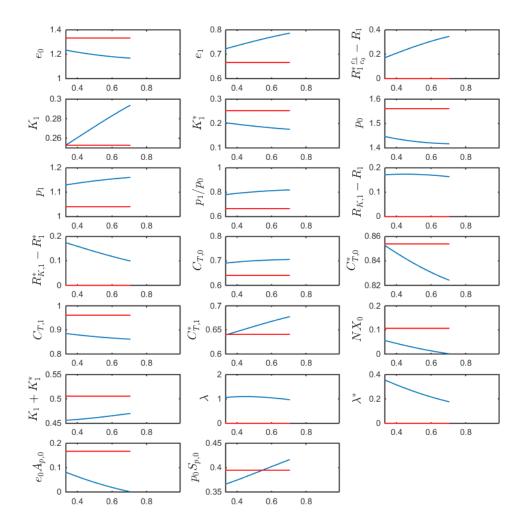


Figure A.5: Effect of an increase in  $\Delta$  on different variables ( $\Delta$  on x-axis). Evolution of the model's equilibrium as the friction parameter in the international credit market increases starting from a point where there are limits to arbitrage in all financial markets.  $\theta = 1/3$ ,  $\theta^* = 1/3$ ,  $\Delta \geq 1/3$ , the remaining parameter values are summarized in Table A.1. The red line represents the value in the frictionless equilibrium.

# B Analytical Solution of the Frictionless Model

In the frictionless model the parameters  $\theta$ ,  $\Delta$  and  $\theta^*$  are either equal to zero or sufficiently low for the banks' incentive constraint not to be binding. In the system of 13 equations derived in Appendix A, equation (88) reduces to  $p_1N_1 = R_1p_0N_0$  since all excess returns are zero and equations (89)-(91) can be replaced by  $R_{k,1} = R_1$ ,  $R_1^* \frac{e_1}{e_0} = R_1$  and  $R_{k,1}^* = R_1^*$ , respectively. In a frictionless environment domestic and foreign banks face no capital constraint therefore we can omit equations (92) and (93), and the variables  $\lambda$  and  $\lambda^*$  and consider the resulting system of 11 equations and 11 unknowns. Under the assumption that  $\delta = 1$ , it is possible to find an analytical solution of the frictionless model. Given that  $Y_{T,0} = Y_{T,0}^*$  and  $L = L^*$ , solving the system of equations yields

$$e_0 = \frac{1+\beta}{1+\beta^*}, \quad e_1 = \frac{\beta^*}{\beta}e_0, \quad K_1 = K_1^* = \gamma_1 Y_{T,0}, \quad p_0 = \gamma_2 \frac{1}{Y_{T,0}}, \quad p_1 = \gamma_3 \frac{1}{Y_{T,1}} = \gamma_3 \frac{1}{K_1^{\alpha} L^{1-\alpha}}$$
 (106)

where  $\gamma_1 \equiv \frac{\alpha\beta(1+\beta^*)+\alpha\beta^*(1+\beta)}{(1+\alpha\beta^*)(1+\beta)+(1+\alpha\beta)(1+\beta^*)}$ ,  $\gamma_2 \equiv \frac{(1+\alpha\beta^*)(1+\beta)+(1+\alpha\beta)(1+\beta^*)}{2(1+\beta^*)}$  and  $\gamma_3 \equiv \frac{\beta(1+\beta^*)+\beta^*(1+\beta)}{2\beta(1+\beta^*)}$ . The exchange rate only depends on the discount factor of home and foreign agents. Investment, and hence production is equally high in both countries and is increasing in endowment of traded goods in the first period  $Y_{T,0}$ . The price of traded goods depends negatively on its supply. The remaining variables of the model can be derived from these 6 variables. Net exports, e.g., are

$$e_0 A_{p,0} = p_0 N X_0 = \frac{\beta - \beta^*}{2(1 + \beta^*)}$$
(107)

# C Equilibrium Equations under Central Bank Intermediation and International Portfolio Flows

Introducing central bank intermediation and international portfolio flows to the baseline model in section 2 leads to the following changes in the system of 13 equations derived in Appendix A: equation (85) is augmented by  $\Pi_{CB,1}$ , the profit of the central bank that is transferred to the domestic household

$$1 = R_1(p_0 Y_{T,0} - p_0 N_0 - 1) + (1 - \alpha) \left(\frac{K_1}{L}\right)^{\alpha} p_1 L + p_1 N_1 + \Pi_{CB,1}$$
(108)

The consolidation of the domestic household's, bank's and central bank's budget constraint (equation (55)) yields

$$e_0 A_{n,0} + e_0 A_{CB,0} + e_0 f - f^* = p_0 N X_0$$
(109)

equation (88) changes to

$$p_1 N_1 = \left(R_{k,1} - R_1\right) p_0 \left(K_1 - S_{CB,0}\right) + \left(R_1^* \frac{e_1}{e_0} - R_1\right) \left(p_0 N X_0 - e_0 f + f^* - e_0 A_{CB,0}\right) + R_1 p_0 N_0$$
(110)

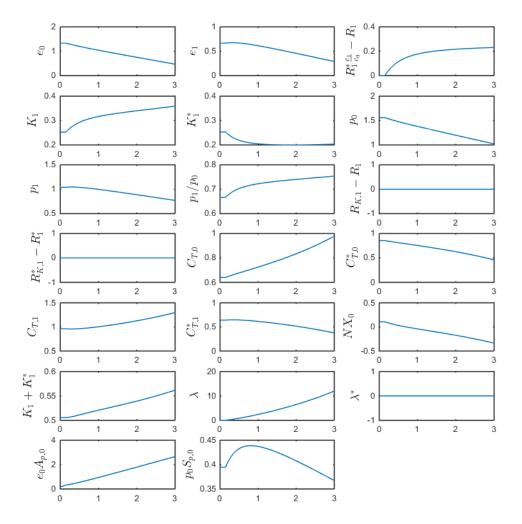
and equation (92) now looks as follows

$$CC = \begin{cases} \frac{\Delta}{\Delta - \frac{1}{R_1} \left( R_1^* \frac{e_1}{e_0} - R_1 \right)} p_0 N_0 \ge \theta p_0 (K_1 - S_{CB,0}) + \Delta (p_0 N X_0 - e_0 f + f^* - e_0 A_{CB,0}) & \text{if} \quad \theta \ge 0, \Delta > 0 \\ \frac{\theta}{\theta - \frac{1}{R_1} \left( R_{k,1} - R_1 \right)} p_0 N_0 \ge \theta p_0 (K_1 - S_{CB,0}) + \Delta (p_0 N X_0 - e_0 f + f^* - e_0 A_{CB,0}) & \text{if} \quad \theta > 0, \Delta \ge 0 \\ \text{no CC}, \lambda = 0 & \text{if} \quad \theta = 0, \Delta = 0 \end{cases}$$

$$(111)$$

where  $p_0NX_0$  is substituted by  $(p_0Y_{T,0}-p_0K_1-1)$ . The KKT conditions change accordingly. Along with these changes, we have to include one additional equation which is the profit of the central bank in period 1 (equation (56)).

The following figures provide a numerical illustration of the model solution under capital inflows, credit easing or foreign exchange interventions. They show the evolution of the model's equilibrium under increasing values for one of these variables using different specifications of the friction parameters and the calibration of Table A.1 for the remaining parameters.



**Figure A.6:** Effect of an increase in  $f^*$  on different variables ( $f^*$  on x-axis). Evolution of the model's equilibrium as capital inflows increase starting from a frictionless point.  $\theta=0, \theta^*=0, \Delta=1/4$ . The remaining parameter values are summarized in Table A.1. Note that the home banks only get credit constrained once  $f^*$  exceeds a certain value.

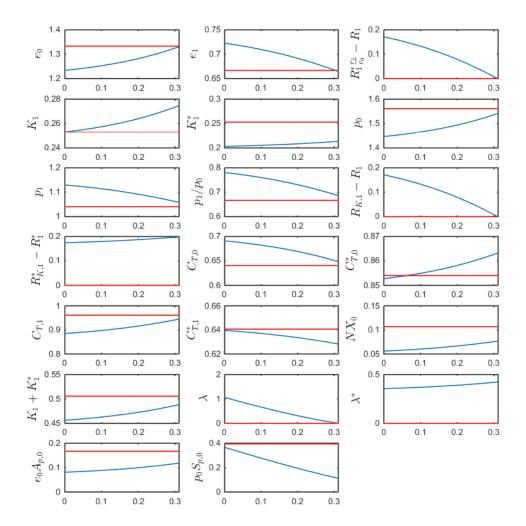


Figure A.7: Effect of an increase in  $S_{CB,0}$  on different variables ( $p_0S_{CB,0}$  on x-axis). Evolution of the model's equilibrium as central bank intermediation in the domestic investment market increases starting from a point where there are limits to arbitrage in all financial markets.  $\theta = 1/3$ ,  $\theta^* = 1/3$ ,  $\Delta = 1/3$ , the remaining parameter values are summarized in Table A.1. The red line represents the value in the frictionless equilibrium.

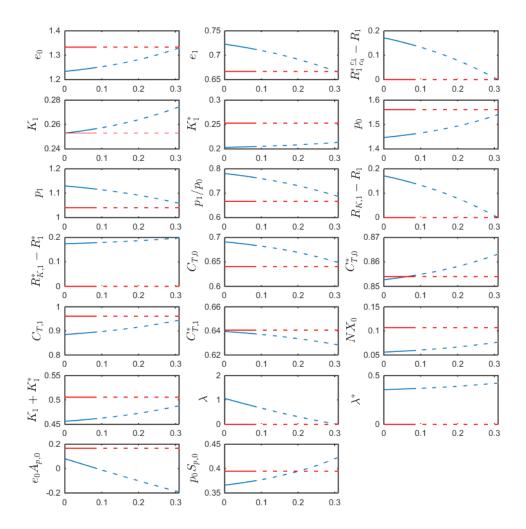


Figure A.8: Effect of an increase in  $A_{CB,0}$  on different variables ( $e_0A_{CB,0}$  on x-axis). Evolution of the model's equilibrium as central bank intermediation in the international credit market increases starting from a point where there are limits to arbitrage in all financial markets.  $\theta = 1/3$ ,  $\theta^* = 1/3$ ,  $\Delta = 1/3$ , the remaining parameter values are summarized in Table A.1. The red line represents the value in the frictionless equilibrium. The dashed part of the lines captures the range of foreign exchange interventions where the latter would require the home banks to go short in foreign bonds in order to fulfill the central bank's demand for these assets and hence covers a part where our model technically is not valid.

# D CPI-based Real Exchange Rate

The exchange rate  $e_t$  is equal to the relative price of the non-tradable goods, i.e. the numéraires, respectively, in our model. To find the CPI-based real exchange rate, we first need to derive the price indices. This is done by replacing  $C_{NT,t}$  and  $C_{T,t}$  in the consumption index by the demand functions resulting from the intratemporal optimization problem (see equations (11) and (12)):

$$C_t = \left(C_{NT,t}^{\chi} C_{T,t}\right)^{\frac{1}{1+\chi}}$$

$$= \left(\left(\frac{\chi}{1+\chi} \left(\frac{1}{P_t}\right)^{-1} C_t\right)^{\chi} \left(\frac{1}{1+\chi} \left(\frac{p_t}{P_t}\right)^{-1} C_t\right)\right)^{\frac{1}{1+\chi}}$$

$$= \frac{\chi^{\frac{\chi}{1+\chi}}}{1+\chi} \left(\frac{1}{p_t}\right)^{\frac{1}{1+\chi}} P_t C_t$$

Hence:

$$P_t = \frac{1+\chi}{\chi^{\frac{\chi}{1+\chi}}} p_t^{\frac{1}{1+\chi}} \tag{112}$$

Similarly, the foreign price index is found to be:

$$P_t^* = \frac{1+\chi}{\chi^{\frac{\chi}{1+\chi}}} p_t^* \frac{1}{1+\chi} \tag{113}$$

It follows that the CPI-based real exchange rate, defined as the ratio of the price indices multiplied by the relative price of the two numéraires, is given by  $^{32}$ :

$$\mathcal{E}_{t} \equiv \frac{P_{t}^{*}}{P_{t}} e_{t} = \left(\frac{p_{t}^{*}}{p_{t}}\right)^{\frac{1}{1+\chi}} e_{t} = \left(\frac{1}{e_{t}}\right)^{\frac{1}{1+\chi}} e_{t}$$

$$= e_{t}^{\frac{\chi}{1+\chi}}$$
(114)

Thus, the exchange rate as we define it in our model is very closely related to the CPI-based real exchange rate. Whenever  $e_t$  is larger (smaller) than 1, this also holds for  $\mathcal{E}_t$ .

# E Proofs: Metzler Diagram

For convenience, Figures 3, 5 and A.1 depict the reaction to a shock where the respective friction parameter passes from being non-binding ( $\lambda = 0$ ) to being binding ( $\lambda > 0$ ). Due to the banks' positive equity capital, this always happens at some strictly positive value of  $\theta$ ,  $\theta^*$  or  $\Delta$ , respectively, denoted by  $\overline{\theta}$ ,  $\overline{\theta^*}$  or  $\overline{\Delta}$ , which represent the highest possible values where the friction parameters

 $<sup>^{32}\</sup>mathcal{E}_t$  is defined to be the price of a foreign consumption bundle expressed in terms of home consumption bundles. Thus, if  $\mathcal{E}_t < 1$ , one consumption bundle in the home country gives more than one consumption bundle in the foreign country.

are still non-binding.<sup>33</sup>

#### E.1 Investment Schedules

The two countries' investment schedules are given by market clearing on the investment markets  $(S_{p,0} = K_1 \text{ and } S_{p,0}^* = K_1^*)$ , by equations (86) and (87), which relate the levels of capital and the (real) returns on the investment securities  $\frac{p_0}{p_1}R_{k,1}$  and  $\frac{p_0^*}{p_1^*}R_{K,1}^*$ , and by equations (7), (26) and (33), which define the relationships between the returns on the investment securities and the return on the bonds  $(R_1 \text{ and } R_1^*, \text{ respectively})$  and result from the banks' optimization problem. Thus, the investment schedules (KK) and  $(KK^*)$  are:

$$(KK) K_1 = \left(\frac{1}{\alpha} \left(\frac{p_0}{p_1} R_{k,1} - (1-\delta)\right)\right)^{\frac{1}{\alpha-1}} L, \text{where} \frac{p_0}{p_1} R_{k,1} = \frac{p_0}{p_1} R_1 \left(1 + \frac{\lambda}{1+\lambda}\theta\right)$$

$$(KK^*) K_1^* = \left(\frac{1}{\alpha} \left(\frac{p_0^*}{p_1^*} R_{K,1}^* - (1-\delta)\right)\right)^{\frac{1}{\alpha-1}} L^*, \text{where} \frac{p_0^*}{p_1^*} R_{K,1}^* = \frac{p_0^*}{p_1^*} R_1^* \left(1 + \frac{\lambda^*}{1+\lambda^*}\theta^*\right)$$

#### • Effect of an increase in $\theta$ :

Using the concept of the total differential, one finds that for a given real interest rate  $\frac{p_0}{p_1}R_1$ , an increase in  $\theta$  has the following effect on home capital:

$$\frac{\mathrm{d}K_{1}}{\mathrm{d}\theta}\Big|_{\frac{p_{0}}{p_{1}}R_{1} \text{ constant}} = \frac{1}{\alpha(\alpha - 1)} \left( \frac{1}{\alpha} \left( \frac{p_{0}}{p_{1}}R_{k,1} - (1 - \delta) \right) \right)^{\frac{2-\alpha}{\alpha-1}} L \frac{\mathrm{d}\left( \frac{p_{0}}{p_{1}}R_{k,1} \right)}{\mathrm{d}\theta} \Big|_{\frac{p_{0}}{p_{1}}R_{1} \text{ constant}}$$

$$= -\frac{1}{\alpha(1-\alpha)} \left( \frac{L}{K_{1}} \right)^{\alpha-2} L \frac{\mathrm{d}\left( \frac{p_{0}}{p_{1}}R_{k,1} \right)}{\mathrm{d}\theta} \Big|_{\frac{p_{0}}{p_{1}}R_{1} \text{ constant}}. \tag{115}$$

The term in front of the final derivative is negative as  $0 < \alpha < 1$ , while the derivative itself is equal to the following expression:

$$\frac{\mathrm{d}\left(\frac{p_0}{p_1}R_{k,1}\right)}{\mathrm{d}\theta}\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} = \frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{p_0}{p_1}R_1\left(1+\frac{\lambda}{1+\lambda}\theta\right)\right)\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}}$$

$$= \frac{p_0}{p_1}R_1\left(\frac{1}{(1+\lambda)^2}\theta\frac{\mathrm{d}\lambda}{\mathrm{d}\theta}\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} + \frac{\lambda}{1+\lambda}\right)$$

$$= \frac{p_0}{p_1}R_1\overline{\theta}\frac{\mathrm{d}\lambda}{\mathrm{d}\theta}\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} > 0, \tag{116}$$

where the last steps follow from the fact that we look at the effect where the friction parameter

<sup>&</sup>lt;sup>33</sup>Intuitively, for very low values of  $\theta$ ,  $\theta^*$  or  $\Delta$ , respectively, the divertable part of a bank's assets will inevitably be lower than the bank's equity capital, which it would loose in case of misbehaviour. Thus, the banks' incentive constraint will not be binding.

passes from being non-binding to being binding ( $\Rightarrow \lambda = 0$  and  $\frac{d\lambda}{d\theta}|_{\frac{p_0}{p_1}R_1 \text{ constant}} > 0$ ), which happens at the strictly positive value  $\theta = \overline{\theta}$ . Altogether, this implies that for a given real interest rate  $\frac{p_0}{p_1}R_1$ , an increase in  $\theta$  leads to a decrease in the level of home capital, which corresponds to a negative shift in the home investment curve.

Likewise, one finds for foreign capital:

$$\frac{dK_{1}^{*}}{d\theta} \Big|_{\substack{p_{0}^{*} R_{1}^{*} \text{ constant}}}^{*} = -\frac{1}{\alpha(1-\alpha)} \left(\frac{L^{*}}{K_{1}^{*}}\right)^{\alpha-2} L^{*} \frac{d\left(\frac{p_{0}^{*}}{p_{1}^{*}} R_{K,1}^{*}\right)}{d\theta} \Big|_{\substack{\frac{p_{0}^{*}}{p_{1}^{*}} R_{1}^{*} \text{ constant}}}^{*}, \qquad (117)$$

where

$$\frac{d\left(\frac{p_0^*}{p_1^*}R_{K,1}^*\right)}{d\theta}\bigg|_{\frac{p_0^*}{p_1^*}R_1^* \text{ constant}} = \frac{d}{d\theta}\left(\frac{p_0^*}{p_1^*}R_1^*\left(1 + \frac{\lambda^*}{1 + \lambda^*}\theta^*\right)\right)\bigg|_{\frac{p_0^*}{p_1^*}R_1^* \text{ constant}} = 0.$$
(118)

Hence, given a foreign real interest rate  $\frac{p_0^*}{p_1^*}R_1^*$ , an increase in  $\theta$  has no effect on foreign investment (remember that  $\theta^* = 0$ ).

### • Effect of an increase in $\theta^*$ :

By the same reasoning as above, one finds:

$$\frac{\mathrm{d}K_1}{\mathrm{d}\theta^*} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} = -\frac{1}{\alpha(1-\alpha)} \left(\frac{L}{K_1}\right)^{\alpha-2} L \frac{\mathrm{d}\left(\frac{p_0}{p_1}R_{k,1}\right)}{\mathrm{d}\theta^*} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}}, \tag{119}$$

where

$$\frac{\mathrm{d}\left(\frac{p_0}{p_1}R_{k,1}\right)}{\mathrm{d}\theta^*} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} = \frac{\mathrm{d}}{\mathrm{d}\theta^*} \left(\frac{p_0}{p_1}R_1 \left(1 + \frac{\lambda}{1+\lambda}\theta\right)\right) \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} = 0. \tag{120}$$

Hence, given a real interest rate  $\frac{p_0}{p_1}R_1$ , an increase in  $\theta^*$  has no effect on home investment (remember that  $\theta = 0$ ).

For the foreign investment curve, one finds:

$$\frac{dK_{1}^{*}}{d\theta^{*}}\Big|_{\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*} \text{ constant}}^{*} = -\frac{1}{\alpha(1-\alpha)} \left(\frac{L^{*}}{K_{1}^{*}}\right)^{\alpha-2} L^{*} \frac{d\left(\frac{p_{0}^{*}}{p_{1}^{*}}R_{K,1}^{*}\right)}{d\theta^{*}}\Big|_{\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*} \text{ constant}}^{*}, \qquad (121)$$

where

$$\frac{\mathrm{d}\left(\frac{p_0^*}{p_1^*}R_{K,1}^*\right)}{\mathrm{d}\theta^*} \bigg|_{\frac{p_0^*}{p_1^*}R_1^* \text{ constant}} = \frac{p_0^*}{p_1^*}R_1^*\overline{\theta^*} \frac{\mathrm{d}\lambda^*}{\mathrm{d}\theta^*} \bigg|_{\frac{p_0^*}{p_1^*}R_1^* \text{ constant}} > 0.$$
(122)

Hence, given a foreign real interest rate  $\frac{p_0^*}{p_1^*}R_1^*$ , an increase in  $\theta^*$  leads to a decrease in the level of foreign capital, which corresponds to a negative shift in the foreign investment curve.

### • Effect of an increase in $\Delta$ :

For the home investment curve, one finds:

$$\frac{\mathrm{d}K_1}{\mathrm{d}\Delta}\Big|_{\frac{p_0}{p_1}R_1 \text{ constant}} = -\frac{1}{\alpha(1-\alpha)} \left(\frac{L}{K_1}\right)^{\alpha-2} L \left.\frac{\mathrm{d}\left(\frac{p_0}{p_1}R_{k,1}\right)}{\mathrm{d}\Delta}\right|_{\frac{p_0}{p_1}R_1 \text{ constant}}, \tag{123}$$

where (remember that  $\theta = 0$ ):

$$\frac{\mathrm{d}\left(\frac{p_0}{p_1}R_{k,1}\right)}{\mathrm{d}\Delta}\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} = \frac{\mathrm{d}}{\mathrm{d}\Delta}\left(\frac{p_0}{p_1}R_1\left(1+\frac{\lambda}{1+\lambda}\theta\right)\right)\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} = 0. \tag{124}$$

Likewise, one finds for foreign capital:

$$\frac{dK_{1}^{*}}{d\Delta} \bigg|_{\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*} \text{ constant}} = -\frac{1}{\alpha(1-\alpha)} \left(\frac{L^{*}}{K_{1}^{*}}\right)^{\alpha-2} L^{*} \frac{d\left(\frac{p_{0}^{*}}{p_{1}^{*}}R_{K,1}^{*}\right)}{d\Delta} \bigg|_{\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*} \text{ constant}}, \qquad (125)$$

where (remember that  $\theta^* = 0$ ):

$$\frac{d\left(\frac{p_0^*}{p_1^*}R_{K,1}^*\right)}{d\Delta}\bigg|_{\frac{p_0^*}{p_1^*}R_1^* \text{ constant}} = \frac{d}{d\Delta}\left(\frac{p_0^*}{p_1^*}R_1^*\left(1 + \frac{\lambda^*}{1 + \lambda^*}\theta^*\right)\right)\bigg|_{\frac{p_0^*}{p_1^*}R_1^* \text{ constant}} = 0.$$
(126)

Hence, given a real interest rate, an increase in  $\Delta$  does not have any effect the level of investment of either country.

### E.2 Savings Schedules

The home country's saving schedule is described by the Euler equation (expressed in terms of traded goods, see equations (7) and (12)), where the households' intertemporal budget constraint (given by  $C_{NT,1} + p_1C_{T,1} = R_1(p_0Y_{T,0} - p_0N_0 + Y_{NT,0} - C_{NT,0} - p_0C_{T,0}) + w_1L + p_1N_1 + Y_{NT,1})$  is used

to eliminate  $p_1C_{T,1}$ :

$$\begin{split} p_0 C_{T,0} &= \frac{p_1 C_{T,1}}{\beta R_1} \\ \Leftrightarrow p_0 C_{T,0} &= \frac{1}{\beta R_1} \left( R_1 (p_0 Y_{T,0} - p_0 N_0 + Y_{NT,0} - C_{NT,0} - p_0 C_{T,0}) + w_1 L + p_1 N_1 + Y_{NT,1} - C_{NT,1} \right) \end{split}$$

Market clearing in the non-traded goods' sector implies that in equilibrium demand and endowment for non-traded goods always have to cancel each other out, and the non-financial firms' technology and optimization behaviour ensure that labor income is a constant share of (nominal) output  $(w_1L = p_1(1-\alpha)Y_{T,1})$ . Finally, the value of the equity capital in period 1,  $p_1N_1$ , is given by equation (64), where by market clearing  $S_{p,0} = K_1$  and  $e_0A_{p,0} = p_0NX_0 = p_0(Y_{T,0} - K_1 - C_{T,0})$ . Thus, the home households' savings schedule (SS) is defined by:

$$p_{0}C_{T,0} = \frac{1}{\beta R_{1}} \left( R_{1}(p_{0}Y_{T,0} - p_{0}C_{T,0}) + p_{1}(1 - \alpha)Y_{T,1} + (R_{k,1} - R_{1}) p_{0}K_{1} + \left( R_{1}^{*} \frac{e_{1}}{e_{0}} - R_{1} \right) p_{0}NX_{0} \right)$$

$$\Leftrightarrow C_{T,0} = \frac{1}{(1 + \beta) \frac{p_{0}}{p_{1}} R_{1}} \left( \frac{p_{0}}{p_{1}} R_{1}Y_{T,0} + (1 - \alpha)Y_{T,1} + \frac{p_{0}}{p_{1}} (R_{k,1} - R_{1}) K_{1} + \frac{p_{0}}{p_{1}} \left( R_{1}^{*} \frac{e_{1}}{e_{0}} - R_{1} \right) NX_{0} \right)$$

$$(127)$$

Likewise, the foreign country's saving schedule  $(SS^*)$  is implicitly given by:

$$C_{T,0}^{*} = \frac{1}{(1+\beta^{*})\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*}} \left(\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*}Y_{T,0}^{*} + (1-\alpha)Y_{T,1}^{*} + \frac{p_{0}^{*}}{p_{1}^{*}}\left(R_{k,1}^{*} - R_{1}^{*}\right)K_{1}^{*}\right)$$
(128)

#### • Effect of an increase in $\theta$ :

The effect of an increase in  $\theta$  on the home country's saving curve can be found by differentiating equation (127), holding  $\frac{p_0}{p_1}R_1$  constant:

$$\frac{\mathrm{d}C_{T,0}}{\mathrm{d}\theta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} = \frac{1}{(1+\beta)\frac{p_0}{p_1}R_1} \left( (1-\alpha)\frac{\partial Y_{T,1}}{\partial K_1} \frac{\mathrm{d}K_1}{\mathrm{d}\theta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} + \frac{\mathrm{d}\left(\frac{p_0}{p_1}R_{k,1}\right)}{\mathrm{d}\theta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} K_1 + \frac{p_0}{p_1} (R_{k,1} - R_1) \frac{\mathrm{d}K_1}{\mathrm{d}\theta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} + \frac{\mathrm{d}\left(\frac{p_0}{p_1}R_1^*\frac{e_1}{e_0}\right)}{\mathrm{d}\theta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} NX_0 + \frac{p_0}{p_1} \left(R_1^*\frac{e_1}{e_0} - R_1\right) \frac{\mathrm{d}NX_0}{\mathrm{d}\theta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} \right)$$

As we consider the case where the countries are initially in the frictionless state, excess returns are zero and the two respective terms disappear. Furthermore, by using equations (7) and (27), we find that  $\frac{d\left(\frac{p_0}{p_1}R_1^*\frac{e_1}{e_0}\right)}{d\theta}\Big|_{\frac{p_0}{p_1}R_1\text{ constant}} = \frac{d}{d\theta}\left(\frac{p_0}{p_1}R_1\left(1+\frac{\lambda}{1+\lambda}\Delta\right)\right)\Big|_{\frac{p_0}{p_1}R_1\text{ constant}} = 0$  (remember that  $\Delta = 0$ ). The two terms that are then still left represent the effect of the

decrease in the households' second-period labour income (due to the lower level of capital) and the higher return on equity capital due to the increase in the excess return on home investment securities. Using equation (115) to replace  $\frac{dK_1}{d\theta}\Big|_{\frac{p_0}{p_1}R_1 \text{ constant}}$  and substituting the

marginal product to capital  $\left(\frac{\partial Y_{T,1}}{\partial K_1} = \alpha \left(\frac{L}{K_1}\right)^{1-\alpha}\right)$ , one finds that they just cancel each other out:

$$\frac{\mathrm{d}C_{T,0}}{\mathrm{d}\theta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} = \frac{1}{(1+\beta)\frac{p_0}{p_1}R_1} \left( (1-\alpha)\frac{\partial Y_{T,1}}{\partial K_1} \frac{\mathrm{d}K_1}{\mathrm{d}\theta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} + \frac{\mathrm{d}\left(\frac{p_0}{p_1}R_{k,1}\right)}{\mathrm{d}\theta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} K_1 \right)$$

$$= \frac{1}{(1+\beta)\frac{p_0}{p_1}R_1} \left( (1-\alpha)\alpha\left(\frac{L}{K_1}\right)^{1-\alpha} \frac{-1}{\alpha(1-\alpha)} \left(\frac{L}{K_1}\right)^{\alpha-2} L \frac{\mathrm{d}\left(\frac{p_0}{p_1}R_{k,1}\right)}{\mathrm{d}\theta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} + \frac{\mathrm{d}\left(\frac{p_0}{p_1}R_{k,1}\right)}{\mathrm{d}\theta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} + \frac{\mathrm{d}\left(\frac{p_0}{p_1}R_{k,1}\right)}{\mathrm{d}\theta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} + \frac{\mathrm{d}\left(\frac{p_0}{p_1}R_{k,1}\right)}{\mathrm{d}\theta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} K_1 \right)$$

$$= \frac{1}{(1+\beta)\frac{p_0}{p_1}R_1} \left( -K_1 \frac{\mathrm{d}\left(\frac{p_0}{p_1}R_{k,1}\right)}{\mathrm{d}\theta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} + \frac{\mathrm{d}\left(\frac{p_0}{p_1}R_{k,1}\right)}{\mathrm{d}\theta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} K_1 \right)$$

$$= 0$$

Hence, for a given real interest rate, an increase in  $\theta$  has no effect on the level of consumption in the home country.

Likewise, by differentiating equation (128), holding  $\frac{p_0^*}{p_1^*}R_1^*$  constant, one finds the effect of an increase in  $\theta$  on the foreign country's saving curve:

$$\frac{\mathrm{d}C_{T,0}^{*}}{\mathrm{d}\theta} \bigg|_{\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*} \text{ constant}} = \frac{1}{(1+\beta^{*})\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*}} \bigg( (1-\alpha) \frac{\partial Y_{T,1}^{*}}{\partial K_{1}^{*}} \frac{\mathrm{d}K_{1}^{*}}{\mathrm{d}\theta} \bigg|_{\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*} \text{ constant}} + \frac{\mathrm{d}\left(\frac{p_{0}^{*}}{p_{1}^{*}}R_{k,1}^{*}\right)}{\mathrm{d}\theta} \bigg|_{\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*} \text{ constant}} K_{1}^{*} + \frac{p_{0}^{*}}{p_{1}^{*}}(R_{k,1}^{*} - R_{1}^{*}) \frac{\mathrm{d}K_{1}^{*}}{\mathrm{d}\theta} \bigg|_{\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*} \text{ constant}} \bigg) = 0.$$

We know from the analysis of the foreign investment curves that the first two expressions in the big brackets are equal to zero (see equations (117) and (118)), and given that the economy is initially in a frictionless state, excess returns are zero as well. Hence, for a given real interest rate, an increase in  $\theta$  has no effect on the foreign country's consumption.

#### • Effect of an increase in $\theta^*$ :

Following the same reasoning as in the case of an increase in  $\theta$ , one finds that both  $\frac{\mathrm{d}C_{T,0}}{\mathrm{d}\theta^*}\Big|_{\frac{p_0}{p_1}R_1 \text{ constant}} = \frac{\mathrm{d}C_{T,0}^*}{\mathrm{d}\theta^*}\Big|_{\frac{p_0^*}{p_1^*}R_1^* \text{ constant}} = 0$ . Thus, an increase in  $\theta^*$  does not lead to a shift in the two countries' saving schedules.

#### • Effect of an increase in $\Delta$ :

The effect of an increase in  $\Delta$  on the home country's consumption, holding  $\frac{p_0^*}{p_1^*}R_1^*$  constant:

$$\frac{\mathrm{d}C_{T,0}}{\mathrm{d}\Delta}\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} = \frac{1}{(1+\beta)\frac{p_0}{p_1}R_1} \left( (1-\alpha)\frac{\partial Y_{T,1}}{\partial K_1} \frac{\mathrm{d}K_1}{\mathrm{d}\Delta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} + \frac{\mathrm{d}\left(\frac{p_0}{p_1}R_{k,1}\right)}{\mathrm{d}\Delta}\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} K_1 + \frac{p_0}{p_1} (R_{k,1} - R_1) \frac{\mathrm{d}K_1}{\mathrm{d}\Delta}\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} + \frac{\mathrm{d}\left(\frac{p_0}{p_1}R_1^*\frac{e_1}{e_0}\right)}{\mathrm{d}\Delta}\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} NX_0 + \frac{p_0}{p_1} \left(R_1^*\frac{e_1}{e_0} - R_1\right) \frac{\mathrm{d}NX_0}{\mathrm{d}\Delta}\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}}\right)$$

Again, excess returns are zero and the two respective terms disappear. From the analysis above, we know that  $\frac{dK_1}{d\Delta}\Big|_{\frac{p_0}{p_1}R_1 \text{ constant}} = 0$  (see equations (123) and (124)). Furthermore, by

using equations (7) and (26), we find that  $\frac{d\left(\frac{p_0}{p_1}R_{k,1}\right)}{d\Delta}\Big|_{\frac{p_0}{p_1}R_1 \text{ constant}} = \frac{d}{d\Delta}\left(\frac{p_0}{p_1}R_1\left(1+\frac{\lambda}{1+\lambda}\theta\right)\right)\Big|_{\frac{p_0}{p_1}R_1 \text{ constant}} = 0$  (remember that  $\theta = 0$ ). The one term that is still left represents the effect of the higher return on equity capital due to the increase in excess returns on foreign assets and equals (using equation (27)):

$$\begin{split} \frac{\mathrm{d}C_{T,0}}{\mathrm{d}\Delta}\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} &= \frac{1}{(1+\beta)\frac{p_0}{p_1}R_1} \left(\frac{\mathrm{d}\left(\frac{p_0}{p_1}R_1^*\frac{e_1}{e_0}\right)}{\mathrm{d}\Delta}\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} NX_0\right) \\ &= \frac{1}{(1+\beta)\frac{p_0}{p_1}R_1} \left(\frac{\mathrm{d}}{\mathrm{d}\Delta}\left(\frac{p_0}{p_1}R_1\left(1+\frac{\lambda}{1+\lambda}\Delta\right)\right)\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} NX_0\right) \\ &= \frac{1}{(1+\beta)\frac{p_0}{p_1}R_1} \left(\frac{p_0}{p_1}R_1 \overline{\Delta}\frac{\mathrm{d}\lambda}{\mathrm{d}\Delta}\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} NX_0\right) \\ &= \frac{1}{(1+\beta)} \left(\overline{\Delta}\frac{\mathrm{d}\lambda}{\mathrm{d}\Delta}\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} NX_0\right) \\ &= \frac{1}{(1+\beta)} \left(\overline{\Delta}\frac{\mathrm{d}\lambda}{\mathrm{d}\Delta}\bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} (Y_{T,0} - K_1 - C_{T,0})\right) > 0. \end{split}$$

As we look at the effect where the friction parameter passes from being non-binding to being

binding, we know that  $\frac{\mathrm{d}\lambda}{\mathrm{d}\Delta}\big|_{\frac{p_0}{p_1}R_1 \text{ constant}} > 0$ . Hence, given a real interest rate, an increase in  $\Delta$  leads to an increase in consumption due to the higher return on home equity capital, which in turn corresponds to a negative shift in the home country's saving curve.

On the other hand, as the foreign banks have no international portfolio and their equity capital is independent of the excess return on foreign international transactions, there is no shift in the foreign country's saving curve:

$$\frac{\mathrm{d}C_{T,0}^{*}}{\mathrm{d}\Delta}\bigg|_{\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*} \text{ constant}} = \frac{1}{(1+\beta^{*})\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*}} \left((1-\alpha)\frac{\partial Y_{T,1}^{*}}{\partial K_{1}^{*}} \frac{\mathrm{d}K_{1}^{*}}{\mathrm{d}\Delta}\bigg|_{\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*} \text{ constant}} + \frac{\mathrm{d}\left(\frac{p_{0}^{*}}{p_{1}^{*}}R_{k,1}^{*}\right)}{\mathrm{d}\Delta}\bigg|_{\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*} \text{ constant}} K_{1}^{*} + \frac{p_{0}^{*}}{p_{1}^{*}}(R_{k,1}^{*} - R_{1}^{*}) \frac{\mathrm{d}K_{1}^{*}}{\mathrm{d}\Delta}\bigg|_{\frac{p_{0}^{*}}{p_{1}^{*}}R_{1}^{*} \text{ constant}}\right) = 0.$$

Again, excess returns in the initial frictionless state are zero, and the first two terms drop out as well:  $\frac{\mathrm{d}\left(\frac{p_0^*}{p_1^*}R_{K,1}^*\right)}{\mathrm{d}\Delta}\Big|_{\frac{p_0^*}{p_1^*}R_1^*\text{ constant}} = \frac{\mathrm{d}}{\mathrm{d}\Delta}\left(\frac{p_0^*}{p_1^*}R_1^*\left(1+\frac{\lambda^*}{1+\lambda^*}\theta^*\right)\right)\Big|_{\frac{p_0^*}{p_1^*}R_1^*\text{ constant}} = 0 \text{ (remember that } \theta^*=0) \text{ and therefore } \frac{\mathrm{d}K_1^*}{\mathrm{d}\Delta}\Big|_{\frac{p_0^*}{p_1^*}R_1^*\text{ constant}} \text{ (see equation (125))}.$