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Abstract

The interaction of macroeconomic variables may change as the nominal short-term interest rates approaches zero. In this paper, we propose an empirical model capturing these changing dynamics with a time-varying parameter vector autoregressive process. State-dependent parameters are determined by a latent state indicator, of which the probability distribution is itself affected by the lagged interest rate. As the interest rate enters the critical zero lower bound (ZLB) region, dynamics between variables and the effect of shocks change. We estimate the model with Bayesian methods and take explicitly into account that the interest rate might be constrained in the ZLB region. We provide an estimate of the latent rate, i.e. the lower interest rate than the observed level which would be state- and model-consistent. The endogenous specification of the state indicator permits dynamic forecasts of the state and the system variables. In an application to Swiss data, we evaluate state-dependent impulse-responses to a risk premium shock identified with sign-restrictions. Additionally, we discuss scenario-based forecasts and evaluate the probability of the system exiting the ZLB region based on the inherent dynamics only.

JEL classification: C3, E3
Key words: Regime switching, time-varying probability, constrained variables.

1 Introduction

The monetary policy instrument of leading central banks is the nominal short-term interest rate. Until recently, the use of this instrument was perceived to face one problem,

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however. Because moderate amounts of money can be stored at relatively low cost, the effective nominal interest rate cannot fall below or not far below zero. Some discussion on the ZLB lower bound took place in the beginning of the century (Woodford 2003, Eggertsson and Woodford 2003; Eggertsson and Woodford 2004, Benhabib et al. 2002 and Auerbach and Obstfeld 2004) and it was recognized that the ZLB might change the functioning of an economy fundamentally. Nevertheless, at the time the ZLB was not perceived to constitute a major problem (Reifschneider and Williams 2000). However, since the outbreak of the financial crisis and the following euro area sovereign debt crisis, the policy rate has remained for a considerably long period of time at the ZLB, in particular in the US, but also in Switzerland. To circumvent the constraint of the ZLB on the policy rate, the US resorted to unconventional monetary policy measures to accommodate the negative effects of the financial crash on the real economy. In Switzerland, the Swiss National Bank (SNB) responded to sharp appreciation pressures by intervening in the foreign exchange market in 2009/2010 and by introducing a minimum exchange rate against the euro in September 2011. In December 2014, the SNB lowered the range for its operational target, i.e. for the three-month libor, into negative territory to between -0.75% and 0.25%. In January 2015, it discontinued the euro-Swiss franc exchange rate floor and moved the target range further into negative territory to between -1.25% and -0.25%.

Still, even if negative policy rates are now being implemented, they are still constrained. As long as cash currency is available, there will be a lower bound at which it will pay off to substitute a deposit account by storage. Moreover, if economic dynamics are changing near this effective ZLB, it is also an open issue whether shocks or interest rate changes have the same effect on e.g. prices, exchange rates and GDP, as when interest rates are out of the ZLB region. One reason for this is that usually, and obviously, the ZLB region is reached because of strongly deteriorating economic and financial conditions. In these periods, uncertainty rises which may change the interest rate sensitivity of economic agents. Moreover, adverse shocks may have different effects if agents expect that the central bank has limited ability to counteract those shocks with further interest rate cuts.

In this paper, we analyze the data by a vector autoregression (VAR) with parameters that are allowed to change when the ZLB becomes binding. A latent state indicator determines the state-specific parameters and the error covariances of the VAR system. The probability distribution of the state indicator depends itself on a covariate which is perceived to be informative on the prevailing state. A natural candidate is the interest rate and currently, we work with this variable. Furthermore, we take into account that the interest rate might be constrained. We are able to provide an estimate of the latent interest rate, i.e. the rate below the observed one which would be state- and model-consistent. The method can be adapted to situations in which several variables are constrained permanently or temporarily. This becomes important in situations in which unconventional monetary policy targets directly prices in specific asset markets, e.g. government bonds, mortgage or currency markets.

We apply our method to analyze the dynamics of Swiss data, namely GDP, the consumer price index (CPI), the effective exchange rate in relation to the nominal interest rate. Taking up the idea of Bäurle and Kaufmann (2014), we analyze how risk premium shocks affecting the exchange rate transmit to prices. We find that risk premium shocks have more persistent effects on prices if the policy rate is constrained, but have only temporary effects if rates can accommodate. The endogenous specification of the state indicator allows to compute dynamic state and variable forecasts. We provide scenario-based forecasts over the period 2014, third quarter, to 2020, third quarter. We find that the system is unlikely to exit the ZLB region as long as appreciation pressures are present.

The next section presents the econometric model and discusses various aspects of the endogenous state probability distribution. Section 3 presents briefly the estimation procedure and describes the computation of the unconditional and the scenario-based forecasts. The results are discussed in section 4 and section 5 concludes. The interested reader finds technical details in the appendix.

2 Econometric model

2.1 Specification

Let y_t be a $N \times 1$ vector of observed variables which follow a vector autoregressive process

$$\begin{aligned} y_t^* &= \mu_t + B_{1t} y_{t-1} + \cdots + B_{pt} y_{t-p} + \varepsilon_t, \ \varepsilon_t \sim N(0, \Sigma_t) \\ y_{1t} &= \max\{y_{1t}^*, \ b\} \\ y_{2t} &= y_{2t}^* \end{aligned} \tag{1}$$

where the variables potentially constrained at a certain bound b are gathered in y_{1t} and the unconstrained variables are collected in y_{2t}. In a standard censored Tobit model (Chib 1992), a positive probability is attached to the constraint b, $P(y_{1t} = b) = P(y_{1t}^* \leq b) > 0$. While y_{1t} is the nominal interest rate in our application, the model is written in general terms such that y_{1t} might be a sub-*vector* of y_t and hence, b might represent a vector of bounds to take into account that other variables might be constrained as well. For example, in the Swiss case, in addition to the interest rate constraint, this could be the 1.20 floor introduced for the euro-Swiss franc exchange rate.

The following two considerations render the modeling approach different from a standard Tobit analysis. The Tobit framework usually applies to a regression relationship with exogenous regressors, in which the dependent variable is censored at a known threshold b. However, in (1) we face an endogenous dynamic relationship, where the current-period variables y_t, including the potentially censored variables y_{1t}, depend on lagged values, and in particular on lagged values of y_{1t}. Second, censored data usually contain the information about which observations are censored, i.e. some observations are constrained to the minimum value b. Our data do not contain this information. For example, the interest rate is always observed, although at a very low value near zero, or even slightly negative, in recent periods. Thus, setting the lower bound equal to $b = 0$ would mean that no observations in fact have been constrained. However, this goes against the view that central banks were constrained in recent times, forcing them to implement unconventional policy measures. This is especially the case for the SNB, which, in reaction to the strong Swiss franc appreciation, may have lowered the interest rate below zero – or lowered it earlier – if it would have been able to do so. Thus, the relevant threshold b might be at a value above, but close to zero. But then, the data do not tell us in which periods central banks were constrained in setting the interest rate.

In light of these two aspects, we might also envisage to use a model in which the interest rate – and the euro-Swiss franc exchange rate – have truncated support with a moving lower truncation threshold given e.g. by the lower bound of the libor target in the SNB's case. We do not pursue this avenue, because we also want to evaluate the extent to which policy is constrained, conditional on all available observed values. If we condition on periods in which we assume the central bank has been constrained, the Tobit framework allows us to form a model-based estimate of the so-called latent interest rate y_{1t}^*, i.e. the interest rate level lower than the threshold which would be consistent with the model and currently observed data.

To address the first issue, we will interpret model (1) as a regression relationship:

$$\begin{aligned} y_t^* &= X_t\beta_t + \varepsilon_t \\ y_{1t} &= \max\{y_{1t}^*,\ b\} \end{aligned} \tag{2}$$

where $X_t = I_N \otimes \left[1,\ y'_{t-1},\ \ldots,\ y'_{t-p}\right]$ and $\beta_t = vec\left([\mu_t,\ B_{1t},\ \ldots,\ B_{pt}]'\right)$. This notation makes explicit that we condition parameter estimation on past *observed* values, i.e. X_t contains all lagged observed values of y_t, also the observed values of very low interest rates. To determine the periods in which the interest rate is constrained, we use $b = 0.25$. This assumption defines the interest rate since 2010 and two observations at the end of the 1970s as being constrained (see figure 2).[1]

To model the time-varying process of the parameters, we rely on a mixture approach

$$\begin{aligned} \beta_t &= \beta_0(1 - I_t) + \beta_1 I_t \\ \Sigma_t &= \Sigma_0(1 - I_t) + \Sigma_1 I_t \end{aligned} \tag{3}$$

where β_0 and Σ_0 represent, respectively, the effects and the error variance structure during times when the interest rate is out of what we call the critical ZLB region, and β_1 and Σ_1 are the parameters prevailing when the interest rate is within the ZLB critical region. The same indicator I_t drives the effects and the error covariance of the system, given that the volatility of constrained variables obviously change and so do the covariances with other variables when the interest rate enters the ZLB critical region. The indicator I_t, which takes on the value 0 or 1, $I_t \in \{0, 1\}$, may be specified ad hoc by defining a priori the periods of very low interest rates. The disadvantage of this procedure is that the relevant threshold for the interest rate at which dynamics change is in fact unknown to the investigator.

Therefore, we assume the indicator I_t to be a latent variable that is to be estimated from the data. A natural indicator of whether I_t turns out 0 or 1, is the departure of the

[1] Bäurle and Kaufmann (2014) take into account another brief episode with interest rates as low as 0.5% in 2003 and 2004.

interest rate from the ZLB in the lagged period. We formulate a probabilistic model for I_t that depends on the lagged interest rate r_{t-1}. To be consistent with model 1, we condition again on past observed values:

$$P\left(I_t = 1 | r_{t-1}, \gamma, \gamma^r\right) = \frac{\exp\left(\gamma^r r_{t-1} + \gamma\right)}{1 + \exp\left(\gamma^r r_{t-1} + \gamma\right)} \tag{4}$$

Alternative specifications might use the inflation rate and the output gap as Taylor rate indicators:

$$\hat{r}_{t-1} = \hat{\mathrm{r}} + \hat{\alpha}_\pi\left(\pi_{t-1} - \pi^*\right) + \hat{\alpha}_y \hat{y}_{t-1}$$

where hats indicate estimates, and $\hat{\mathrm{r}}$, $(\pi_t - \pi^*)$ and $\hat{y}_t$ represent, respectively, the long-run average interest rate, the deviation of the inflation rate from target and the output gap. This specification has advantages when the interest rate reaches the zero lower bound. An increasing inflation rate and an increasingly positive output gap indicate rising interest rates again away from the zero lower bound towards regime $I_t = 0$.

2.2 Some considerations on the probability function

As it stands, the transition function (4) is not identified. To ensure that $I_t = 1$ will indicate periods in which the interest rate is in the critical ZLB region, we set the state-identifying restriction $\gamma^r < 0$.

Moreover, we call the parametrization (4) the *implicit threshold parametrization* because an estimate allows to recover the threshold after having estimated the model. The threshold is defined as the level of r_t at which the state probability equals 0.5. For example, if the interest rate were expressed in percentage terms, if $\gamma^r = -1$ and $\gamma = 0.5$, the threshold level would lie at $-\gamma/\gamma^r = 0.5\%$. To estimate the model, we will use parametrization (4) because, by additionally implementing a two-layer data augmentation step, the non-linear model in γ^r and γ becomes linear. This allows us to draw from full conditional distributions. However, the drawback of parametrization (4) is a high correlation between γ^r and γ, as we will see below.

The usual probability parametrization, which we call the *explicit threshold parametrization* includes explicitly the threshold $\tilde{\gamma}$:

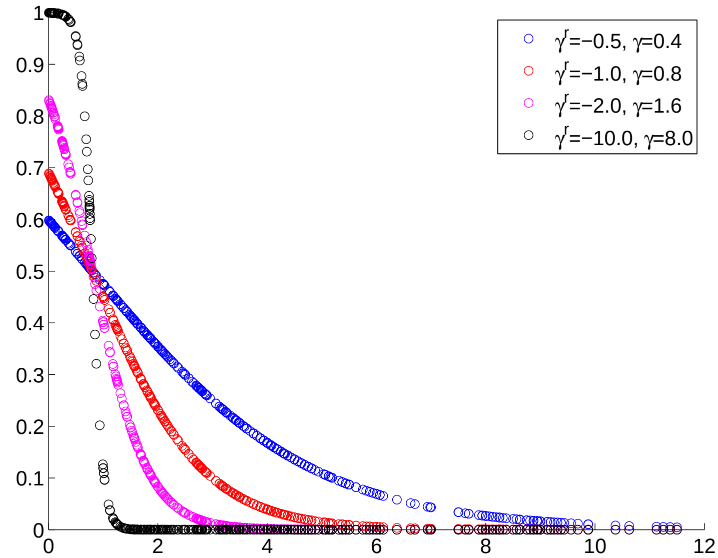
$$P\left(I_t = 1 | r_{t-1}, \gamma, \gamma^r\right) = \frac{\exp\left(\gamma^r\left(r_{t-1} - \tilde{\gamma}\right)\right)}{1 + \exp\left(\gamma^r\left(r_{t-1} - \tilde{\gamma}\right)\right)} \tag{5}$$

Note that from an estimate of (4), in case $\gamma^r \neq 0$, we can also retrieve the threshold level $\tilde{\gamma}$,

$$\begin{aligned} -\gamma^r \tilde{\gamma} &= \gamma \\ \tilde{\gamma} &= -\gamma/\gamma^r \end{aligned} \tag{6}$$

From (6) it becomes clear why, for a given threshold, γ^r and γ are highly negatively correlated. For a given threshold, γ increases with the sensitivity of the state probability with respect to the covariate, γ^r. Figure 1 illustrates this point. The figure plots values for the short term interest rate against the state probability obtained for various γ^r, assuming a threshold level of 0.8%. As $-\gamma^r$ increases, the probability function approaches a step function. To keep the threshold unchanged, γ increases respectively by the same factor as γ^r.

Figure 1: State probability $P(I_t = 1|r_t, \gamma^r, \gamma)$ for various sensitivities γ^r, where γ is adjusted such to keep the threshold level at 0.8%.



The relationship between both parameterizations can be used to include information into the prior distribution for the parameters of the state probabilities. We may have some idea of an upper and of a lower bound for $\widetilde{\gamma}$. For example, $\widetilde{\gamma}$ is certainly well below 10%, is probably below 1%, and perhaps between 0.5% and 1.5%. So, let the upper and lower bound on $\widetilde{\gamma}$ be $\overline{\gamma}$ and $\underline{\gamma}$, respectively, such that $\underline{\gamma} < \widetilde{\gamma} \leq \overline{\gamma}$.

This implies $0 \leq \underline{\gamma} \leq -\frac{\gamma}{\gamma^r} \leq \overline{\gamma}$, or

$$-\gamma^r \underline{\gamma} \leq \gamma \leq -\gamma^r \overline{\gamma} \tag{7}$$

This puts an upper and a lower bound on γ since $\gamma^r < 0$. The prior for (γ, γ^r) is expressed with these inequalities in place:

$$\pi(\gamma, \gamma^r) = N(g_0, G_0)\, 1(\gamma^r < 0)\, 1\left(-\gamma^r \underline{\gamma} \leq \gamma \leq -\gamma^r \overline{\gamma}\right). \tag{8}$$

3 Estimation and forecasting

3.1 Estimation

To describe the estimation of model (2)-(4) in a concise way, we introduce additional notation. While the vector y_t represents the vector of observed variables, see specification

(2), the vector y_t^* represents the augmented data vector, which contains all uncensored variables, $y_t^* = (y_{2t}', y_{1t}^{*\prime})'$. Bold faced objects gather all observations of a data vector or a latent variable, e.g. $\boldsymbol{y} = \{y_t | t = 1, \dots, T\}$, and similarly for $\boldsymbol{y}^*$ and $\mathbf{I}$. We gather all latent values of the censored variables in $\boldsymbol{y_1^*} = \{y_{1t}^* | t \in \boldsymbol{t}^*\}$, $\boldsymbol{t}^* = \{\tau | y_{1\tau} \leq b, \tau = 1, \dots, T\}$. The parameters are included in $\theta = \{\beta_k, \Sigma_k, \boldsymbol{\gamma} | k = 0, 1, \boldsymbol{\gamma} = (\gamma^r, \gamma)\}$, and the augmented parameter vector adds the latent variables to θ, $\vartheta = \left\{\theta, \boldsymbol{y_1^*}, \mathbf{I}\right\}$.

We apply Bayesian Markov chain Monte Carlo (MCMC) methods to estimate the model. By combining the likelihood with the prior distribution, we obtain the conditional posterior

$$\pi(\vartheta | \boldsymbol{y}) \propto f(\boldsymbol{y}^* | \mathbf{X}, \mathbf{I}, \theta)\, \pi(\mathbf{I} | \mathbf{r}, \theta)\, \pi\left(\boldsymbol{y_1^*}\right) \pi(\theta) \tag{9}$$

To obtain a sample from (9), we draw from the posterior of

(i) $\mathbf{I}$, $\pi(\mathbf{I} | \boldsymbol{y}^*, \mathbf{X}, \mathbf{r}, \theta)$
(ii) $\boldsymbol{y_1^*}$, $\pi\left(\boldsymbol{y_1^*} | \boldsymbol{y_2}, \mathbf{X}, \mathbf{I}, \theta\right) 1(\boldsymbol{y_1^*} \leq b)$
(iii) $\boldsymbol{\gamma}$, $\pi(\boldsymbol{\gamma} | \mathbf{r}, \mathbf{I})\, 1(\gamma^r < 0)\, 1\left(-\gamma^r \underline{\gamma} \leq \gamma \leq -\gamma^r \overline{\gamma}\right)$
(iv) the rest of the parameters, $\theta_{-\boldsymbol{\gamma}}$, $\pi(\theta_{-\boldsymbol{\gamma}} | \mathbf{X}, \boldsymbol{y}^*, \mathbf{I})$

All posterior distributions are standard distributions. Given that there is no state persistence, in step (i) we can sample $\mathbf{I}$ in one draw from a discrete distribution. Conditional on observed values $\boldsymbol{y_2}$, $\mathbf{I}$ and the model parameters, we draw $\boldsymbol{y_1^*}$ from a truncated normal distribution. To derive the posterior of the parameters governing the state distribution, we condition on two layers of data augmentation (see Frühwirth-Schnatter and Frühwirth 2010 and Kaufmann 2015). In the first layer, we obtain a linear model with non-normal error terms, which relates the difference in latent state utilities to the interest rate effect on the state probability. In a second layer, we approximate the exponential error distribution by a mixture of M normals. Conditional on the differences in latent utilities and the components of the mixtures, the posterior of $\boldsymbol{\gamma}$ is normal. We draw from the normal posterior truncated to the region where the parameters restrictions derived in (8) are fulfilled. The posterior distribution of the remaining parameters in (iv), are normal and inverse Wishart, respectively, for β_k and Σ_k, $k = 0, 1$. The interested reader finds a detailed derivation of the likelihood, and the prior and posterior distributions as well in appendix C.

3.2 Forecasting

The model can be used to obtain forecasts over the forecast horizon H, $h = 1, \dots, H$. To obtain draws from the unconditional posterior predictive distribution at each horizon h:

$$\pi(y_{T+h} | y_T) \propto \prod_{j=1}^{h} \pi(y_{T+j} | y_{T+j-1}, I_{T+j})\, \pi(I_{T+j} | y_{T+j-1}) \tag{10}$$

we produce dynamic forecasts and simulate for $j = 1, \dots, h$

1. $I_{T+j}^{(l)}$ from $\pi\left(I_{T+j}^{(l)} | r_{T+j-1}^{(l)}, \boldsymbol{\gamma}^{(l)}\right)$, with $r_T^{(l)} = r_T$.

2. $y^{(l)}_{T+j}$ from $\pi\left(y^{(l)}_{T+j}|y^{(l)}_{T+j-1}, I^{(l)}_{T+j}, \theta^{(l)}\right) \sim N\left(m^{(l)}_{T+j}, \Sigma_{I^{(l)}_{T+j}}\right)$
with $m^{(l)}_{T+j} = X^{(l)}_{T+j}\beta_{I^{(l)}_{T+j}}$, $y^{(l)}_T = y_T$ and $X^{(l)}_{T+1} = X_{T+1}$.

for each draw (l) out of the posterior $\pi(\vartheta|\boldsymbol{y})$.

We may also produce so-called conditional forecasts, which would reflect specific scenarios. In all examples, step 2 above is adjusted appropriately. For example

2. (i) keep the mean forecast of the interest rate at or above the last rate, i.e. restrict the predictive distribution to:

$$\pi\left(y_{T+h}|y_T, (m_{1,T+1}, \ldots, m_{1,T+h}) = y_{1T}\right) \text{ or } \pi\left(y_{T+h}|y_T, (m_{1,T+1}, \ldots, m_{1,T+h}) \geq y_{1T}\right)$$

The second conditional forecast is implemented as follows. At each step j, $j = 1, \ldots, h$, we set $m^{(l)}_{1,T+j} = \max\left\{X^{(l)}_{1,T+j}\beta_{1,I^{(l)}_{T+j}}, y_{1T}\right\}$.

2. (ii) implement a (mean) path for a variable i over a certain period of time, say $h = 1, \ldots, 4$, (e.g. lower the interest rate to -1% for one year):
Simulate first variable i, $y^{(l)}_{i,T+h}$, from $N\left(m_{i,T+h}, \Sigma_{i,I^{(l)}_{T+h}}\right)$, where $m_{i,T+h}$ is pre-specified.
Then, conditional on $y^{(l)}_{i,T+h}$, simulate all other variables, $y^{(l)}_{-i,T+h}|y^{(l)}_{i,T+h}$, from $N\left(m_{-i|i,T+h}, \Sigma_{-i|i,I^{(l)}_{T+h}}\right)$, the moments of which are given by the moments of the implied normal conditional predictive distribution.

2. (iii) a combination of the two. Here we apply 2.(ii), except that $y^{(l)}_{i,T+h}$ refers to sub-vector i of y_{T+h}, which is generated from the joint predictive distribution with restricted means.

4 Results

4.1 Specification

To illustrate the method, we estimate a level VAR for four Swiss variables, namely GDP, the consumer price index (CPI), the 3-month libor and the trade-weighted effective exchange rate. We use quarterly data covering the period 1974, first quarter, to 2014, third quarter. As already mentioned, the state-identifying restriction $\gamma^r < 0$ defines $I_t = 1$ as indicating the periods in which the interest rate enters the ZLB critical region. We additionally induce the threshold, i.e. the level of the interest rate at which $P(I_t = 1) = 0.5$, to lie in the interval $[\underline{\gamma}, \overline{\gamma}] = [0.5, 1.5]$. Hence the prior mean for the threshold is 1.0. In the ZLB critical region, we define interest rate levels at or below 0.25 as those being constrained, i.e. $b = 0.25$.

The specification of the prior hyperparameters, $\pi(\vartheta)$, completes the Bayesian setup in (9):

1. We assume a Minnesota type prior for the VAR parameters β_k, $k = 0, 1$ (Doan et al. 1984; Bańbura et al. 2010).

2. The scale S_0 of the Wishart distribution is proportional to the residuals variance of state-specific univariate autoregressive processes, $S_{0k,ii} \propto \sigma^2_{i,I_t}$.

3. A relatively informative prior on γ^r is used to obtain a steep shape of the transition function (see figure 1). This is necessary also due to the relative low number of observations near the ZLB.

$$\begin{pmatrix} \gamma^r \\ \gamma \end{pmatrix} \sim N\left(\begin{bmatrix} -10 \\ 0 \end{bmatrix}, \mathrm{diag}\,(0.01, 6.25)\right) 1\,(\gamma^r < 0)\, 1\,(-0.5\gamma^r \leq \gamma \leq -1.5\gamma^r)$$

4. For $\boldsymbol{y}_1^*$, we work with a diffuse prior, $\pi\left(\boldsymbol{y}_1^*\right) \propto 1(\boldsymbol{y}_1^* \leq b)$.

4.2 Model inference

To estimate the model, we iterate $M = 10,000$ times over the sampling steps (i)-(iv) listed in section 3.1 and retain the last $8,000$ to compute posterior moments. Figure 2 plots the interest rate and the inflation rate along with the mean posterior probabilities of state 2 in yellow. The estimate is able to discriminate clearly between the two states. State 2 is estimated to prevail also at the end of the 1970s, a period where the Swiss franc was also subject to appreciation against the German mark and where therefore, interest rates were also decreased to a then all-time low. The horizontal line indicates the threshold level at 1.5%, inferred from the parameter estimates of the transition function.

Figure 2: Annual inflation rate (red) and interest rate (blue). Mean posterior probability (yellow) of state 1. The periods during which the interest rate is defined to be constrained are those in which the interest rate lies in the shaded area ($b \leq 0.25$). The horizontal line indicates the inferred threshold level 1.5% at which $P\left(I_t = 1\right) = 0.5$.

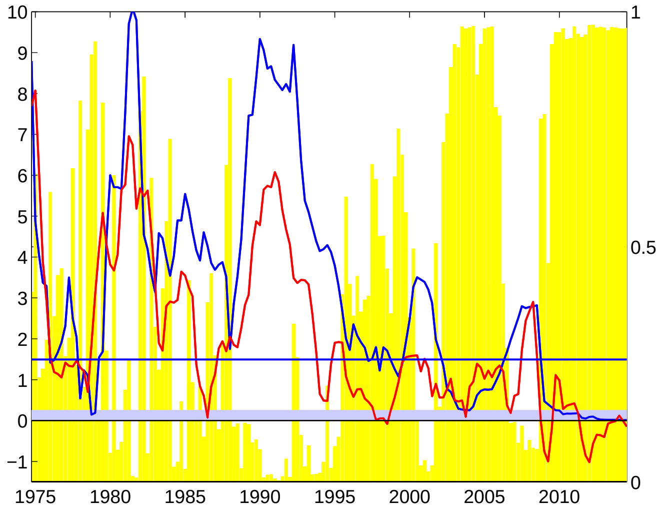
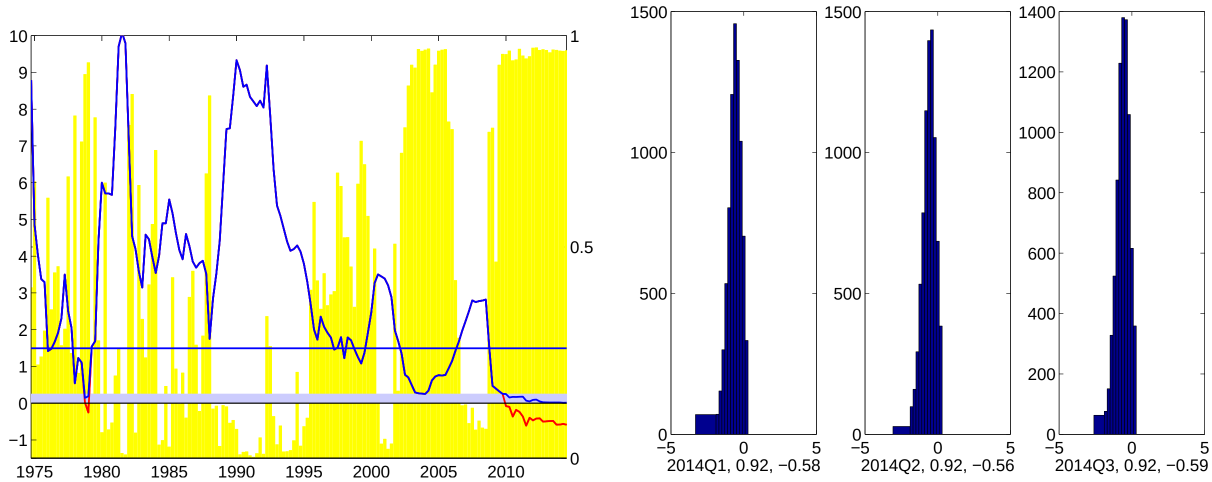


Figure 3: Left-hand: Observed interest rate and model-based estimate of $\boldsymbol{y}_1^* < b$ (red). Mean posterior probability of state 1 (yellow). Right-hand: histogram of $y_{1t}^* < b$.



Period t, $P\left(y_{1t}^* < \min_t\{y_{1t}\}\right)$, median $\left(y_{1t}^*\right)$

The shaded area below $b = 0.25$ indicates in which periods the interest rate is thought to be constrained. On the left-hand side in figure 3, the observed interest rate is plotted along with the median estimate of the latent observations. Compared with the end of the 1970s, the ZLB on the interest rate appears to bind more strongly. Up to the end of the sample, the median of the latent interest rate decreases to nearly -0.6%. The right-hand histograms in figure 3 show additionally that over 90% of the sampled period-specific latent interest rates, $y_{1t}^* < b$, were lower than the all-time minimum observed value for the interest rate, $\min_t\{y_{1t}\}$.

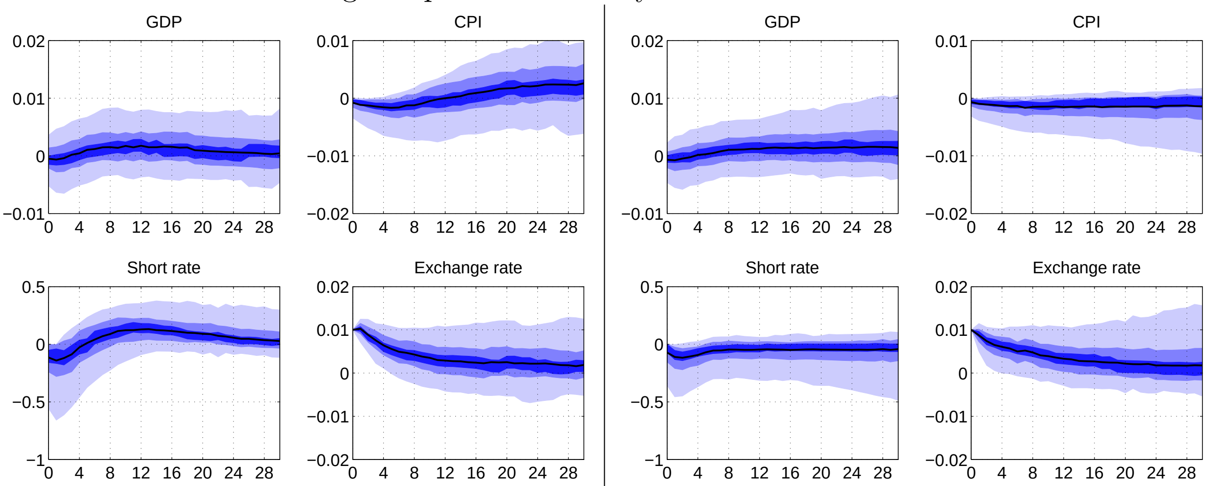
To document that dynamics change when the interest rate enters the critical ZLB region, we plot impulse responses to a structural shock identified as a risk-premium shock. Monetary policy can counter-act the effects of a risk-premium shock, which effects an appreciation for a small open economy, by lowering the interest rate. Obviously, this reaction will be constrained if the interest rate is already very low. As a consequence, the short-term and the long-term pass-through effects on prices will also differ in the two situations (see also Bäurle and Kaufmann 2014). To obtain structural identification, we impose sign restrictions on the impact and next period responses of the variables as shown in table 1 (Arias et al. 2014). A risk-premium shock is expected to appreciate the currency. In a small open economy, the pass-through should lead to a decrease in prices. Monetary policy can counteract the effects by lowering the interest rate. The response of GDP is not restricted. All responses are left unrestricted after the first two periods. This allows to infer whether the medium-term and the long-term effects differ between the two states.

The state-specific impulse responses to a risk-premium shock are plotted in figure 4. The responses are normalized to correspond to 1% appreciation in the exchange rate. Although the density intervals are quite large, tendencies are recognizable. In the short-term, we observe that there is obviously more leeway for the interest rate to decrease transitorily in state 1 ($I_t = 0$). The response of GDP is not restricted and broadly insignificant.

Table 1: Sign restrictions on the impact and next period responses of the variables.

	Reaction in			
Shock to	GDP	CPI	Short rate	exchange rate
Risk-premium	–	↓	↓	↑

Figure 4: Impulse responses to a risk premium shock identified by sign restrictions and normalized to a corresponding 1% appreciation shock. The black line is the median response, the areas decreasing in shades correspond to, respectively, the 25%, the 50% and the 80% interval of highest posterior density.



(a) $I_t = 0$ (b) $I_t = 1$

Nevertheless, the median shows a positive transitory effect, likely initiated by the decrease in the interest rate. The negative pass-through to prices is transitory and after two years, level-reversion takes place. In particular the long-run effects on prices are very different in state 2 ($I_t = 1$). Given that in the ZLB region the interest rate cannot react as strongly to the risk-premium shock, the initial negative pass-through to prices remains permanent. Or in other words, a transitory risk-premium shock translates into a permanent effect on the price level. Although long-run cross-country relationships are not modelled explicitly, we know that for a given real exchange rate, permanent negative effects on the price level induce further long-run nominal appreciation on the currency. Overall, these results are consistent with those presented in Bäurle and Kaufmann (2014).

4.3 Forecasts

The model estimate is used to answer the following questions. Where does the system drift to if the mean interest is observed to fall to -1% in the first quarter of the forecast horizon, remains at this level for one year but is left unconstrained afterwards? What is the probability of exiting the ZLB critical region and under what economic conditions does this happen in this scenario? These questions may be relevant against the backdrop of the SNB's recent decision to introduce negative interest rates. However, it is important to recognize that our scenario does not implement a *policy experiment*, i.e. it does not provide an estimate of the *causal* impact of a decrease in interest rates to -1%. It merely describes the economic conditions *consistent with* an average interest rate at -1% for one year.

The forecast horizon is 6 years, $H = 24$. The sample from the forecasting density (10) is obtained by producing dynamic forecasts using all posterior parameter draws. Figure 5 displays the forecasts we obtain if the mean interest rate stays at a level of -1% for one year from 2014Q4 onwards , this corresponds to the second setting in 2.(ii) in section 3.2. Over the whole forecasting period, the mean interest rate remains quite stable in this scenario. However, there is a relevant chance (12%) for the system to exit the ZLB region. GDP growth and inflation are low on average, but still positive (0.7% and 0.2%, respectively). This overall stable development is accompanied by further appreciation pressures.

Figure 6 plots the forecasts of those 12% paths that finally exit the ZLB region again. On average, GDP growth and inflation reach respectively, 1.4% and 1.1%, and we observe that they are accelerating over the forecast period. At the end of the forecast horizon annual GDP growth reaches 3% and inflation 2.3%. At the same time, the appreciation trend is broken. On average, depreciation amounts to 0.1%, while towards the end of the forecast horizon it increases to 3.1%. Thus, the results indicate that economic conditions have to improve quite substantially to make an exit from the ZLB regime happening endogenously. In contrast to the previous results, an exit from the ZLB region would be accompanied by a depreciation of the Swiss franc.

Figure 5: Left-hand: Forecast distribution conditional on a mean interest rate lowered to -1% for 1 year. The black line is the median forecast, the areas decreasing in shades correspond to, respectively, the 25%, the 50% and the 80% interval of highest forecast density. The vertical line denotes the end of the sample, 2014Q3. Right-hand: Mean forecast probability of $I_{T+h} = 1$.

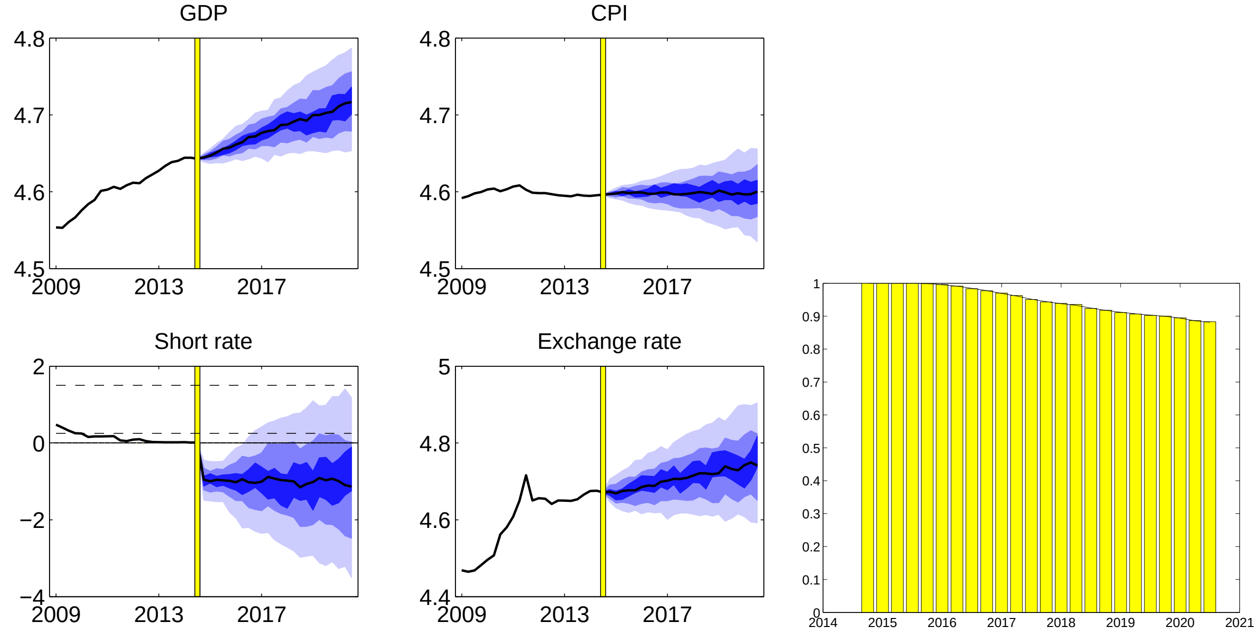
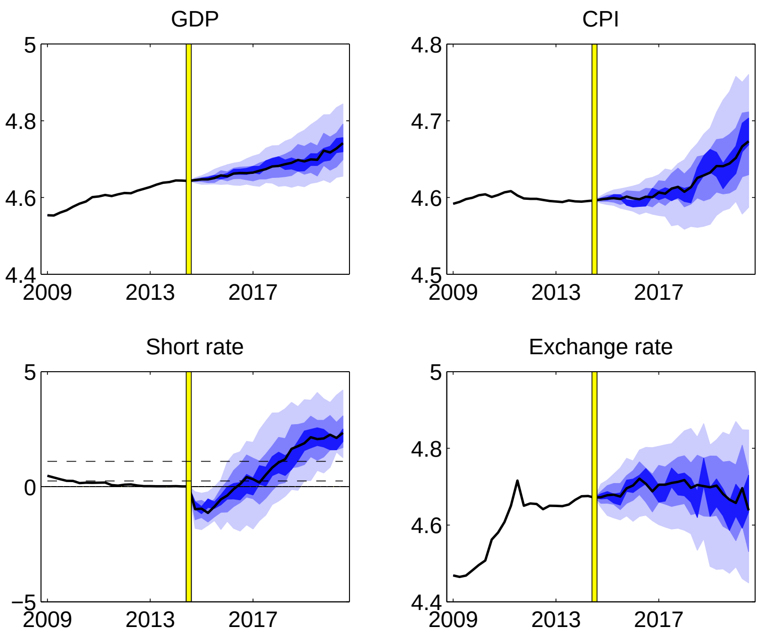


Figure 6: Conditional forecast distribution for those paths that exit the ZLB region (mean 12% probability), $I_{T+H} = 0$. The black line is the median forecast, the areas decreasing in shades correspond to, respectively, the 20%, the 50% and the 80% interval of highest forecast density. The vertical line denotes the end of the sample, 2014Q3.



5 Conclusion

In the present paper, we propose to capture changing dynamics between variables near the ZLB with the use of a nonlinear model. A latent state indicator determines the changes in parameters and error covariances of a VAR model. The logit model for the state probability does itself depend on a covariate, which is perceived to significantly indicate whether the system is away or in the so-called critical ZLB region. Currently, we work with the lagged interest rate level as covariate in the probability function. It is obvious, that other variables determining the policy stance like GDP growth or the inflation rate could also be used as covariates. For a small open economy, another alternative could be to include a monetary condition index, which determines the monetary stance by a weighted average of the interest rate and the exchange rate. The specification of the VAR model takes into account that in the ZLB region, the interest rate might be a constrained variable. The estimation of the model then provides us with an inference on the latent rate, i.e. the lower than observed level of the interest rate which would be state- and model-consistent.

We setup a model for four Swiss variables, namely GDP, CPI, the libor and a trade-weighted effective exchange rate. We estimate it within a Bayesian framework, which allows to handle the situation of few observations near the ZLB. Also, we can input subjective information into the specification of prior distributions. For example, a notion for an upper and a lower bound of the interest rate at which we think that dynamics may change, a prior notion on the threshold value, can be included into the prior of the parameters of the state probability distribution. The results show that dynamics indeed change when the interest rate enters the ZLB region. The impulse response analysis gives evidence that transitory risk-premium shocks which correspond to a 1% appreciation in the exchange rate translate into a permanent negative price level effect when the interest rate is in the ZLB region. This differs from the normal situation, in which the negative price level effect is transitory, too.

The endogenous specification of the state probability distribution allows to dynamically forecast the state and the VAR system into the future. In particular, we can evaluate the probability with which the system can exit the ZLB region based on its own dynamics. We find that there is a relevant chance to exit the ZLB in coming years. However, this is unlikely to happen as long as the Swiss franc is under appreciation pressures.

The model used in the present paper can be extended in various ways. The model for a small open economy would be completed by including a set of foreign, exogenous variables. Additional scenarios could then be evaluated, like the reaction to a further increase or decrease in the foreign policy rate, or a protracted recovery abroad. Another avenue would be to model explicitly long-run common trending behavior among the variables. An issue that is not addressed in the paper is how to identify a monetary policy shock in the ZLB region. Further research will address these extensions.

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A Distributional properties of censored and uncensored variables

Given the normality assumption on ε_t, model (2) defines a joint normal distribution for the variables $y_t^* = [y_{1t}^{*\prime}, y_{2t}^{\prime}]^{\prime}$, where y_{2t} gathers the uncensored variables.

$$\begin{bmatrix} y_{1t}^* \\ y_{2t} \end{bmatrix} |X_t, I_t, \theta \sim N\left(\begin{bmatrix} m_{1I_t} \\ m_{2I_t} \end{bmatrix}, \begin{bmatrix} \Sigma_{11,I_t} & \Sigma_{12,I_t} \\ \Sigma_{21,I_t} & \Sigma_{22,I_t} \end{bmatrix} \right) \tag{11}$$

where $\theta = \{\beta_k, \Sigma_k, \gamma^r, \gamma | k = 0, 1\}$ represents the model parameters and $m_{iI_t} = X_{it}\beta_{i,I_t}$ and Σ_{ij,I_t} are obtained by gathering the corresponding rows in (2) and by partitioning accordingly the moment matrices. This allows to express the joint observation density $f(y_t^*)$ as the product of a marginal and a conditional density, $f(y_t^*|\cdot) = f(y_{1t}^*|y_{2t}, \cdot)f(y_{2t}|\cdot)$, where:

$$\begin{aligned} f(y_{2t}|\cdot) &= N\left(m_{2I_t}, \Sigma_{22,I_t}\right) = N\left(X_{2t}\beta_{2I_t}, \Sigma_{22,I_t}\right) && (12) \\ f(y_{1t|2}^*|\cdot) = f(y_{1t}^*|y_{2t}, \cdot) &= N\left(m_{1I_t|2}, M_{1I_t|2}\right) && (13) \\ \text{with} & \\ m_{1I_t|2} &= m_{1I_t} + \Sigma_{12,I_t}\Sigma_{22,I_t}^{-1}\left(y_{2t} - m_{2I_t}\right) && (14) \\ M_{1I_t|2} &= \Sigma_{11,I_t} - \Sigma_{12,I_t}\Sigma_{22,I_t}^{-1}\Sigma_{21,I_t} && (15) \end{aligned}$$

The factoring of $f(y_t^*|\cdot)$ partitions the joint distribution into two parts and allows to implement a normal regression model for the unconstrained variables and a conditional censored normal regression model for the constrained variables

$$\begin{bmatrix} y_{1t|2} \\ y_{2t} \end{bmatrix} |X_t, I_t, \theta \sim N\left(\begin{bmatrix} m_{1I_t|2} \\ m_{2I_t} \end{bmatrix}, \begin{bmatrix} M_{1I_t|2} & 0 \\ 0 & \Sigma_{22,I_t} \end{bmatrix} \right) 1\left(y_{1t} \geq b\right) \tag{16}$$

B Bayesian framework

B.1 Likelihood

Define the number N_j, $j = 1, 2$, which indicates the number of, respectively, censored and uncensored variables.

Conditional on $\mathbf{I}$ and using (16), the data likelihood can be factorized

$$\begin{aligned} f\left(\boldsymbol{y}|\mathbf{X}, \mathbf{I}, \theta\right) &= \prod_{t=p+1}^{T} f\left(y_t|X_t, \beta_{I_t}, \Sigma_{I_t}\right) 1\left(y_{1t} \geq b\right) && (17) \\ &= \prod_{t=p+1}^{T} f\left(y_{1t}|y_{2t}, X_{1t}, \beta_{1I_t}, \Sigma_{11,I_t}\right) 1\left(y_{1t} \geq b\right) f\left(y_{2t}|X_{2t}, \beta_{2I_t}, \Sigma_{22,I_t}\right) && (18) \end{aligned}$$

From (12), the period t density contribution is multivariate normal

$$f\left(y_{2t}|X_{2t},\beta_{2I_t},\Sigma_{22,I_t}\right)=(2\pi)^{-N_2/2}\left|\Sigma_{22,I_t}\right|^{-1/2}\times \exp\left\{-\frac{1}{2}\left(y_{2t}-X_{2t}\beta_{2I_t}\right)'\Sigma_{22,I_t}^{-1}\left(y_{2t}-X_{2t}\beta_{2I_t}\right)\right\} \quad (19)$$

and the period t contribution of censored variables is

$$f\left(y_{1t}|y_{2t},X_{1t},\beta_{1I_t},\Sigma_{11,I_t}\right)1\left(y_{1t}\geq b\right)=\Phi\left(M_{1I_t|2}^{-1/2}\left(b-m_{1I_t|2}\right)\right)^{1(y_{1t}=b)}\times \left|M_{1I_t|2}\right|^{-1/2}\phi\left(M_{1I_t|2}^{-1/2}\left(y_{1t}-m_{1I_t|2}\right)\right)^{1(y_{1t}>b)} \quad (20)$$

where $\Phi(z)$ for the $N_1\times 1$ vector z,

$$\Phi(z)=\int_{-\infty}^{z_1}\dots\int_{-\infty}^{z_{N_1}}\left|M_{1I_t|2}\right|^{-1/2}\phi(z)dz_1\dots dz_{N_1},$$

denotes the cdf and ϕ the pdf (see (19)) of the standard (multivariate) normal distribution.

The likelihood of the complete data factorizes

$$f\left(\boldsymbol{y}^*|\mathbf{X},\mathbf{I},\theta\right)=\prod_{t=p+1}^{T}f\left(y_{1t}^*|y_{2t},X_{1t},\beta_{1I_t},\Sigma_{11,I_t}\right)f\left(y_{2t}|X_{2t},\beta_{2I_t},\Sigma_{22,I_t}\right) \quad (21)$$

where the moments of the conditional and marginal normal observation densities are given in, respectively, (13)-(15) and (12).

B.2 Prior distributions

To complete the Bayesian setup, we specify the prior density of the state indicator $\mathbf{I}$:

$$\pi\left(\mathbf{I}|\mathbf{r},\gamma,\gamma^r\right)=\prod_{t=p+1}^{T}\pi\left(I_t|r_{t-1},\gamma,\gamma^r\right) \quad (22)$$

The prior for the censored variables is assumed to be diffuse, $\pi\left(\boldsymbol{y}_1^*\right)\propto 1(\boldsymbol{y}_1^*\leq b)$. We might also work with a proper prior distribution, restricted to the latent area, $\pi\left(\boldsymbol{y}_1^*\right)\sim N\left(0,\kappa I\right)1(\boldsymbol{y}_1^*\leq b)$ with κ some real number.

Finally, the prior specification on the model parameters completes the Bayesian setup. We assume independent priors:

$$\pi\left(\theta\right)=\pi\left(\gamma,\gamma^r\right)\prod_{k=0}^{1}\pi\left(\beta_k\right)\pi\left(\Sigma_k\right) \quad (23)$$

The prior for the parameters governing the state probability distribution includes a state-identifying restriction and additional information on the threshold level, $\pi\left(\gamma,\gamma^r\right)=N\left(g_0,G_0\right)1\left(\gamma^r<0\right)1\left(-\gamma^r\underline{\gamma}\leq\gamma\leq-\gamma^r\overline{\gamma}\right)$.

The priors on β_k are independent normal, with variance structure implied by Minnesota priors, $\pi\left(\beta_k\right)=N\left(0,\underline{V}\right)$, $k=0,1$. For Σ_k, we implement an inverse Wishart distribution $W^{-1}(S_0,s_0)$ with scale S_0 and prior degrees of freedom s_0.

C Posterior distributions

To obtain draws from the posterior

$$\pi\left(\vartheta|\boldsymbol{y}\right) \propto f\left(\boldsymbol{y}^{*}|\mathbf{X},\mathbf{I},\theta\right)\pi\left(\mathbf{I}|\mathbf{r},\theta\right)\pi\left(\boldsymbol{y}_{\mathbf{1}}^{*}\right)\pi\left(\theta\right)$$

we sample iteratively from the posterior of

1. the state indicator, $\pi\left(\mathbf{I}|\boldsymbol{y}_{\mathbf{1}}^{*},\mathbf{X},\mathbf{r},\theta\right)$. Given that there is no state dependence in the state probabilities, we are able to sample the states simultaneously. We update the period t prior odds $P\left(I_t=1\right)/P\left(I_t=0\right)=\exp\left(\gamma^r r_{t-1}+\gamma\right)$ to obtain the posterior odds

$$P\left(I_t=1|\cdot\right)/P\left(I_t=0|\cdot\right)=\frac{f\left(y_t^{*}|X_t,\beta_1,\Sigma_1\right)\exp\left(\gamma^r r_{t-1}+\gamma\right)}{f\left(y_t^{*}|X_t,\beta_0,\Sigma_0\right)},\ t=p+1,\ldots,T$$

   We sample $T-p$ uniform random variables U_t, and set $I_t=1$ if

$$P(I_t=1|\cdot)/\left(P(I_t=0|\cdot)+P(I_t=1|\cdot)\right)\geq U$$

2. the censored variables, $\pi\left(\boldsymbol{y}_{\mathbf{1}}^{*}|\boldsymbol{y}_{\mathbf{2}},\mathbf{X},\mathbf{I},\theta\right)1(\boldsymbol{y}_{\mathbf{1}}^{*}\leq b)$. Conditional on $\mathbf{I}$ and the observed variables, and given a diffuse prior, the moments of the posterior normal distribution $\pi\left(\boldsymbol{y}_{\mathbf{1}}^{*}|\cdot\right)$ are given by (13)-(15). We sample from this distribution truncated to the region $\boldsymbol{y}_{\mathbf{1}}^{*}\leq b$.

3. the parameters of the state distribution,$\pi\left(\boldsymbol{\gamma}|\mathbf{r},\mathbf{I}\right)\,1\left(\gamma^r<0\right)1\left(-\gamma^r\underline{\gamma}\leq\gamma\leq-\gamma^r\overline{\gamma}\right)$. First, we introduce two layers of data augmentation, which renders the non-linear, non-normal model into a linear-normal model for the parameters (Frühwirth-Schnatter and Frühwirth 2010):

   - We express the state distribution in relative terms, as the difference between the latent state utilities

$$\begin{aligned}\varpi_t &= I_{1t}^u-I_{0t}^u\\ &= \gamma^r r_{t-1}+\gamma+\epsilon_t,\ \epsilon_t\sim\text{Logistic}\end{aligned}$$

     where

$$\begin{aligned}I_{1t}^u &= \gamma^r r_{t-1}+\gamma+\nu_{1t},\ \text{and } I_{0t}^u=\nu_{0t}\\ &\quad\text{with } \nu_{kt}\text{ i.i.d. Type I EV}\end{aligned}$$

   - We approximate the Logistic distribution by a mixture of normals with M components, $\boldsymbol{R}=(R_1,\ldots,R_T)$. Conditional on the latent relative state utilities $\boldsymbol{\varpi}$ and the components, we obtain a normal posterior distribution, $N(g,G)$ with moments:

$$\begin{aligned}G &= \left(G_0^{-1}+\sum_{t=p+1}^{T}Z_t'Z_t/s_{m_t}^2\right)^{-1}\\ g &= G\left(G_0^{-1}g_0+\sum_{t=p+1}^{T}Z_t'\varpi_t/s_{m_t}^2\right)\end{aligned}$$

where $Z_t = [r_{t-1}, 1]'$ and $s^2_{m_t} = s^2_m$ is the variance of the mixture components R_t, see table 2 in Frühwirth-Schnatter and Frühwirth (2010).

To implement the restrictions on $\boldsymbol{\gamma}$ according to (7), we partition the posterior appropriately:

$$\pi\left(\gamma^r, \gamma|\cdot\right) \sim N\left(\begin{bmatrix} g_1 \\ g_2 \end{bmatrix}, \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}\right)$$

Then we first sample $\gamma^{r,(mc)}$ from $N(g_1, G_{11})1\left(\gamma^r < 0\right)$ and then sample γ from the truncated conditional posterior (Robert 2009 or Botev 2016):

$$\gamma|\gamma^r = \gamma^{r,(mc)} \sim N\left(g_2^c, G_2^c\right) 1\left(-\gamma^r \underline{\gamma} \leq \gamma \leq -\gamma^r \overline{\gamma}\right)$$

with moments

$$\begin{aligned} g_2^c &= g_2 - G_{21} G_{11}^{-1}\left(\gamma^{r,(mc)} - g_2\right) \\ G_2^c &= G_{22} - G_{21} G_{11}^{-1} G_{12} \end{aligned}$$

4. the rest of the parameters, $\pi\left(\theta_{-\boldsymbol{\gamma}}|\mathbf{X}, \boldsymbol{y}_{\mathbf{1}}^{*}, \mathbf{I}\right)$. Conditional on $\mathbf{I}$ and the augmented data $\boldsymbol{y}^{*}$, the model in (2) becomes linear normal. The posterior distribution of the regression parameters and of the error variance are then, respectively, normal and inverse Wishart, the moments of which can be derived in the usual way.