Financing Government Expenditures Optimally

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Abstract

In a simple cash-credit model, I study the effects of the combination of costly tax collection and tax evasion on fiscal and monetary policy for optimal resource allocation. Allowing the informal sector to use cash more intensively than the formal sector, I compute the optimal interest and tax rates for eleven OECD countries to finance their exogeneously given government spending. A comparison of the actual and optimal interest rates reveals that tax collection costs and tax evasion together can partly explain the cross-country differences in monetary policy, also rationalizing deviations from the Friedman Rule in the long-run.

JEL Classification Numbers: E63, H21, H26

Keywords: Optimal Interest Rates, Tax Collection Costs, Tax Evasion, Friedman Rule, Cash-intensive Informal Sector, Inflationary Finance.

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1 Introduction

Collecting 1 dollar in taxes cost 1.60 cents in Portugal in year 2000, almost four times as much as in the US, according to the OECD (2004). In fact, Portugal is no exception, there are countries with higher tax administration costs, and the Internal Revenue Services is not operating on a modest budget either[1]. On the other hand, increasing the money supply by 1 dollar is virtually costless for any central bank. New printed banknotes are mostly used to replace the worn out bills — a service provided to the public to facilitate trade, so that the printing cost of an additional dollar remains negligible[2].

Furthermore, in many countries, including the industrialized ones, there is a nontrivial fraction of the economy evading taxes but engaged in legal economic activities otherwise. This sector uses cash intensively because it helps to avoid transactions being traced back, and is therefore also referred to as the cash economy in the literature. Faced with a costly tax collection system and a cash economy together, one could argue that governments might find printing money a cheap and easy way of "collecting" inflation tax. Thus relying on inflationary finance could be optimal from the governments' perspective.

The purpose of this paper is to consider tax evasion and costly tax collection in a dynamic general equilibrium framework and to quantitatively analyze their impact on monetary and fiscal policy for optimal resource allocation. Using a simple cash-credit model I calculate long-run optimal interest and tax rates to finance an exogeneously given government spending for 11 OECD countries. I find that tax evasion and costly consumption taxes can together explain some of the variation in monetary and fiscal policies across countries. In fact, the model predicts optimal interest rates that are quite close to actual ones for many of the countries in the sample, thus rationalizing the observed deviations from the Friedman Rule. Thus I also confirm the robustness of previous estimates of optimal interest rates reported in Yeşin (2004) where costly income taxes and tax evasion were modeled. Comparison of optimal and actual seigniorage revenues reveals that at least some governments were inflating optimally to create seigniorage revenue for financing

[1] E.g., according to the official numbers, IRS budget was about than 2.5 times the budget of the FBI in 1998.

[2] Nonetheless, according to the Federal Reserve Board, approximately 8.23 billions of banknotes were printed in 2005, half of which were in 1 and 5 dollar denominations. Less than 10% of the new notes were denominated as 50 and 100 dollars. It cost about 458 million dollars to print banknotes worth 142.2 billions of dollars, i.e. about 0.003 cents per dollar.

their expenditures — albeit they were not always on the "correct" side of a seigniorage Laffer Curve.

I model the consumption goods sector following Nicolini (1998) with the additional assumption that consumption taxes are costly to collect for the government. This is a crucial assumption since it changes the government's problem and hence its optimal policy decision in a significant way. For example, in this model, the optimal interest rate for the US is computed to be higher than what Nicolini (1998) estimates[3], and closer to the long-run average. Using the recently available data set on tax collection costs in OECD (2004), I show that the contribution of costly collection procedures to the optimal level of interest rates is significant, even though the costs are not too high.

There are several reasons why consumption taxes are used in the model instead of — or in combination with — income taxes. First, the model is simple and tractable and can give the main intuition also without income taxes. Second, no specific assumption is needed regarding which type of tax is more costly to collect. Third, modeling the choice of working in the formal or informal sector is avoided here. This might be essential, since there are no estimates of the elasticity of substitution between formal and informal sector across countries[4]. In fact, in a related paper with costly income taxes and tax evasion, Yeşin (2004) finds that the optimal interest rates are sensitive to the choice of this parameter in the estimations. Thus with this model, the robustness of the previous results can be checked as well. Fourth, although most countries rely on a combination of income and consumption taxes as instruments, consumption taxes have become more and more important throughout the world in the recent years due to their simplicity to administer and to collect. For example, 29 out of 30 OECD countries as of 2006 have a Value Added Tax (VAT) or a Goods and Services Tax (GST)[5]. Thus the model here can be seen as the case when the governments rely solely on consumption taxes. And last but not least, in this model cash usage can be assumed to be more intensive in the informal sector

[3]The optimal interest rate for the US in Nicolini (1998) and in this paper are not directly comparable since the baseline values both papers try to match in the calibrations are different. Nicolini (1998) matches data for 1982 when the size of the government was higher than its long-run average. This variable, in turn, increases the optimal interest rates.

[4]This parameter is estimated only for Canada by Lemieux, Fortin, and Frechette (1994).

[5]The US is the only OECD member country without a federal value added type tax on consumer expenditure, although most of its states administer sales taxes.

than in the formal sector to reflect the common perception.

In the model and/or estimations I abstract from several things, including time-variation of interest rates, determinants of government spending, tax collection for redistributive purposes, tax burden on the consumers, the interdependence of tax collection costs and tax evasion, dollarization, the commitment problem of the government, tax collection costs incurred by the consumers, and the decision on working in the formal or informal sector.

Table 1 lists tax collection cost data reported in OECD (2004) for the sample for 2000-2002. Although there is some time-variation of the fraction of net tax revenues needed to finance the tax authority's budget, the values look stable over time and seem to be driven largely by country specifics[6]. In the estimations I approximate the efficiency of the tax collection system for each country as one minus the average unit cost of tax collection and assume a linear tax collection cost function[7]. Data on average interest rates, seigniorage revenues, the size of the government, and tax revenues as a fraction of the output during 1976-2000 are given in Table 2. The variation in these variables across countries is remarkable. Table 2 also reports the size of the informal sector in 1989-1993 for the sample estimated by Schneider and Enste (2000) and Öğünç and Yılmaz (2000). I report the range implied by their estimates and use the midpoint in my estimations. Figures 1, 2, and 3 demonstrate the main hypotheses of this paper that for countries with higher tax collection costs and/or larger informal sector, the average interest rates and seigniorage revenues have been indeed higher in the past. For more detailed info on the data used here see Yeşin (2004).

In the estimations I vary the relative cash intensity of the informal sector, as done in Nicolini (1998). First assuming that cash usage in the formal and informal sector is the same, I calculate a lower bound for optimal interest rates and an upper bound for optimal tax rates. Then I let the informal sector to be twice and five times more cash intensive to see how sensitive the estimates are. Although optimal interest rates increase with the informal sector's cash usage, the effect is significantly different from country to country depending on the country's specific situation including the size of its government and the size of its informal sector. Since the cash intensity

[6]These costs refer to both income and consumption tax collection. Unfortunately, separate data is not available to assess which tax is more costly to collect.

[7]A linear tax collection cost function is assumed for three reasons: 1. for the sake of simplicity, 2. annual IRS budget and tax collection data during 1976-2000 does not suggest otherwise, 3. to make the new results regarding optimal interest rates comparable with the existing literature.

of the informal sector has not been estimated empirically before, one should bear in mind that the relative cash intensity of the informal sector is probably not the same in each country.

Another important aspect is whether taxes on the cash good are paid in cash or not. Since there is no commonly used convention in the literature, I consider both versions of the cash-in-advance constraints for the model, and report my results in the main body of the paper and in the appendix. The conclusion that inflationary finance might have been chosen optimally by some governments remain supported in both versions of the economy.

This paper is organized as follows: Section 2 reviews the related literature, Section 3 describes the model, Section 4 gives the quantitative results and Section 5 concludes. The results for the alternative specification of the cash-in-advance constraint are given in the Appendix.

2 Related Literature

The Friedman Rule states that for optimal resource allocation in the economy, net nominal interest rates should be zero. This 'Ramsey problem' result has been proven to be robust for a wide range of dynamic stochastic general equilibrium models with income or consumption taxes. The results are summarized in Chari, Christiano, and Kehoe (1996), Correia and Teles (1999), and Chari and Kehoe (1999), among others. These models assume that the collection of taxes is not costly for the government in the same way as money printing is not. Neither is there tax evasion modeled. Thus optimality requires that money should not be taxed because it can be considered as an intermediate input from the consumer's perspective.

There have been some attempts in the literature to explain the deviations from the Friedman Rule for single countries, mainly by assuming tax evasion. For example, Nicolini (1998) calculates optimal interest rates for the US and Peru in a cash-credit model assuming the presence of an informal sector within the economy. The optimal interest rate for Peru in the year 1982 is calculated to be between 9% and 19%, depending on the cash intensity of the informal sector, even when the informal sector is assumed to be quite large, namely 40% of the official economy. Since in 1982 nominal interest rates in Peru were around 60%, he concludes that massive tax evasion in Peru does not explain the deviation of the interest rates from the Friedman Rule. However, he notes that optimal interest rates depend not only on the size of the informal sector, but also largely on the size of the government. In fact, Peru with a relatively small government

should not have needed to rely much on inflationary finance.

Cavalcanti and Villamil (2003), on the other hand, assume the presence of a similar informal sector in the economy and prove that the Friedman Rule is not optimal. Using the US economy as a baseline, they find that for alternative calibrations the annual optimal inflation can range from 0% to 22%; however, they do not take a stand on how to interpret their numbers and compare them with actual interest rates.

In a shopping time model de Fiore (2000) shows that there are conditions under which the Friedman Rule is still optimal despite the presence of tax collection costs. She estimates optimal annual interest rates for the US at less than 1% even when the tax collection generates losses as high as 20% of the revenue.

Yeşin (2004) is the first attempt to systematically analyze cross-country differences in monetary policy taking into account the fact that income taxes imply collection costs whereas fiat money can be printed without cost in the Ramsey Problem framework, and also the first to estimate tax collection costs for a variety of countries. In that study, I find that for some countries the optimal and actual interest rates are quite close to each other, and I also show that optimal interest rates are sensitive to the parameter choice for the elasticity of substitution between formal and informal labor. Since there is no literature estimating this parameter for a variety of countries, no further progress is possible for this line of enquiry. On the other hand, in this paper the informal sector is assumed to be in the final goods sector, thus the decision on whether to work for the informal or formal sector does not need to be explicitly modeled; and the parameter choice issue can be avoided. Furthermore, this papers makes use of a newly available data set on tax collection costs in OECD (2004) improving the precision of the estimations.

The link between currency demand and the informal sector (in terms of tax pressure) was first analyzed by Cagan (1958) and then developed by Tanzi (1980) and Tanzi (1983). On page 313 Cagan (1958) describes the link between currency demand and the informal sector in the US as follows:

> "*The use of currency to conceal taxable transactions was probably higher during and after the war, primarily because income tax rates were raised substantially early in World War II and have not been appreciably reduced since. Income received, held, and spent without prior deposit in a bank usually defies detection. A tax on income*

thus leads some people to receive income and make expenditures as far as possible without the use of checks."

Therefore it is important to consider cash usage in the informal sector when estimating optimal interest rates.

3 Model

This simple cash-credit model with informal and formal sectors is based on Nicolini (1998). There is a continuum of consumption goods indexed by the interval [0,1]. The representative agent's preferences over consumption goods and leisure can be represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \Big[U\big(c_t(z)\big) dz \Big] - V(n_t) \right\} \tag{1}$$

where U is increasing and concave, and V is increasing and convex; $c_t(z)$ is consumption of good z at time t, n_t is time allocated to work at time t, and $0 < \beta < 1$ is the discount factor.

There is a constant returns to scale technology that can transform one unit of labor into one unit of output. There are four types of consumption goods: informal credit, informal cash, formal cash and formal credit goods. More specifically, the unit interval of the consumption goods can be partitioned as

$$[0,1] = \overbrace{\underbrace{[0,a]}_{\text{credit goods}} \cup \underbrace{(a,a+b]}_{\text{cash goods}}}^{\text{informal sector}} \cup \overbrace{\underbrace{(a+b,a+b+c]}_{\text{cash goods}} \cup \underbrace{(a+b+c,1]}_{\text{credit goods}}}^{\text{formal sector}}$$

where a, b, and c are positive real numbers such that their sum is lower than 1. I assume that all goods in $[0, a+b]$ are traded in the informal market, which is not registered with the government. The goods in $(a+b, 1]$ are traded in the formal market. Moreover, all goods in $(a, a+b+c)$ must be traded using cash, i.e. they are cash goods. The goods in $[0, a] \cup (a+b+c, 1]$ are credit goods. Here a, b, c, and $d = 1-a-b-c$ represent the relative sizes of the four different goods markets. In the quantitative part, I will assume that the informal sector may make greater use of cash than the formal sector.

Each period, the government prints money, issues one-period bonds that pay a gross nominal interest rate R, and collects consumption taxes for the goods traded in the formal sector to finance

a given stream of government spending. While collecting the consumption taxes, however, the government incurs tax collection or enforcement costs.

Let the tax collection cost function be linear in the total tax revenue:

$$\phi(T) = (1-\kappa)T \tag{2}$$

where T is the total nominal value of taxes collected by the government, and κ is the efficiency parameter of the tax system[8] such that $0 \leq \kappa \leq 1$.

Since the government can only collect consumption taxes for the goods traded in the formal sector of this economy $\big[\text{for } z \in (a+b,1)\big]$, the government's budget constraint in period t is

$$R_t B_t + p_t^g g_t + (1-\kappa)\tau_t \int_{a+b}^{1} \big[p_t(z)c_t(z)\big]dz = M_{t+1} - M_t + B_{t+1} + \tau_t \int_{a+b}^{1} \big[p_t(z)c_t(z)\big]dz \tag{3}$$

where $\{g_t\}_t$ is a given stream of government consumption expenditures, τ_t is the uniform consumption tax rate in the formal sector, M_t is the nominal money supply, B_t is the nominal bond supply, p_t^g is the price of government consumption expenditures, and $p_t(z)$ is the price of consumption good z at time t.

The consumer faces a cash-in-advance constraint for the cash good. Since there is no common convention in the literature I will consider two cases in order to ensure that the analysis is complete. The first cash-in-advance constraint I consider is identical to what Nicolini (1998) assumes. In (4) the consumer does not have to pay the tax on the formal cash good in cash. But I also consider the other case, (4'), where the consumption tax on the formal cash good must be paid in cash. These two constraints are:

$$M_t \geq \int_{a}^{a+b+c} [p_t(z)c_t(z)]\,dz \tag{4}$$

$$M_t \geq \int_{a}^{a+b} \big[p_t(z)c_t(z)\big]dz + (1+\tau_t)\int_{a+b}^{a+b+c} \big[p_t(z)c_t(z)\big]dz \tag{4'}$$

The results in the main body of this paper are based on the cash-in-advance constraint (4). However, I do the whole analysis with (4') as well and report my findings in the Appendix.

The period budget constraint of the consumer is

$$M_{t+1} + B_{t+1} + \int_{0}^{a+b} \big[p_t(z)c_t(z)\big]dz + (1+\tau_t)\int_{a+b}^{1} \big[p_t(z)c_t(z)\big]dz = p_t^n n_t + M_t + B_t R_t \tag{5}$$

[8]Equivalently, $1-\kappa$ is the unit cost of tax collection. This is the same tax collection cost function as in Yeşin (2004).

where p_t^n is the price of labor, n_t.

Hence, the consumer's problem is to maximize (1) subject to (4) $\big($or (4')$\big)$, (5), and initial wealth and non-negativity conditions. To rule out Ponzi schemes, I let real bond holdings be bounded above and below, which will not be binding in equilibrium.

An allocation is denoted by $s = \Big\{\{c_t(z)\}_{z\in[0,1]}, n_t, M_t, B_t\Big\}_{t=0}^{\infty}$, the price system is denoted by $q = \Big\{\{p_t(z)\}_{z\in[0,1]}, p_t^g, p_t^n, R_t\Big\}_{t=0}^{\infty}$, and government policy is $\pi = \{\tau_t\}_{t=0}^{\infty}$.

Definition 1. *A competitive equilibrium is a government policy, π, a price system, q, and an allocation, s, such that*

1. given π and q, the allocation s solves the representative consumer's utility maximization problem.

2. given π and q, the government's budget constraint (3) is satisfied for all t.

Given the technology, it must be true that at any equilibrium the prices of consumption goods, labor and government spending are equal to one other. That is

$$p_t(z) = p_t^n = p_t^g = p_t \quad \text{for all } z \in [0,1] \quad \text{for all } t$$

Therefore the resource constraint on this economy at time t is

$$n_t = g_t + \int_0^1 c_t(z)dz + (1-\kappa)\tau_t\Big[\int_{a+b}^1 c_t(z)dz\Big] \tag{6}$$

The last term in (6) denotes resources wasted during the tax collection process.

It is assumed that the government has a commitment technology and can bind itself to the optimal policy at time zero.

Definition 2. *A Ramsey equilibrium is an allocation, s, for the consumer and a government policy, π, such that the government policy solves the problem:*

$$\max_{\{c_t(z)_{z\in[0,1]}, n_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}\beta^t\Bigg\{\int_0^1\Big[U\big(c_t(z)\big)dz\Big] - V(n_t)\Bigg\}$$

subject to the constraint that there exists $\{M_t, B_t, \tau_t\}_{t=0}^{\infty}$ such that given $\pi = \{\tau_t\}_{t=0}^{\infty}$, $s = \Big\{\{c_t(z)\}_{z\in[0,1]}, n_t, M_t, B_t\Big\}_{t=0}^{\infty}$ is a competitive equilibrium allocation.

Since there are no differences among the goods in a given market, at any equilibrium, the consumption levels in a given market will be identical. From now on, I will drop the notation with

z whenever possible, and use the following variables instead: Denote the credit informal goods $\big[\text{when } z \in (0,a)\big]$ as y_t^I, the cash informal goods $\big[\text{when } z \in (a,a+b)\big]$ as x_t^I, the cash formal goods $\big[\text{when } z \in (a+b,a+b+c)\big]$ as x_t^F, and finally the credit formal goods $\big[\text{when } z \in (a+b+c,1)\big]$ as y_t^F.

It should also be noted that for consumers to be willing to hold bonds, the rate of return must be at least equal to the rate of return on fiat money. Hence, the gross nominal interest rate must be greater or equal to 1.

The following proposition enables us to simplify the Ramsey Problem for the economy specified above.

Proposition 1. *[Ramsey Allocation] The consumption and labor allocations in a competitive equilibrium satisfy:*

$$n_t = g_t + ay_t^I + bx_t^I + cx_t^F + (1-a-b-c)y_t^F + (1-\kappa)\Big[\frac{U'\big(y_t^F\big)}{V'(n_t)} - 1\Big]\big(cx_t^F + (1-a-b-c)y_t^F\big) \quad (7)$$

$$\sum_{t=0}^{\infty}\beta^t\Big[ay_t^I U(y_t^I) + bx_t^I U(x_t^I) + cx_t^F U(x_t^F) + (1-a-b-c)y_t^F U(y_t^F) - n_t V'(n_t)\Big] = 0 \quad (8)$$

$$U'\big(x_t^I\big) \geq U'\big(y_t^I\big) \quad (9)$$

$$U'\big(y_t^I\big) = V'(n_t) \quad (10)$$

$$U'\big(y_t^F\big) - U'\big(x_t^F\big) = U'\big(y_t^I\big) - U'\big(x_t^I\big) \quad (11)$$

$$\frac{U'\big(y_t^F\big)}{V'(n_t)} \geq 1 \quad (12)$$

$$x_t^I, y_t^I, y_t^F, x_t^F, n_t \geq 0 \quad (13)$$

Furthermore, the allocations that satisfy the above equations can be decentralized as a competitive equilibrium.

Proof. : Although straightforward, the proof is tedious and has therefore been omitted here. See Yeşin (2004) for an analogous proof using income taxes. □

The intuitive interpretations of the above constraints are as follows: (7) resembles the resource constraint where the consumer's first order condition for the marginal rate of substitution between consumption of formal credit good and labor has been substituted as the tax rate. Constraint (8) is called the implementability constraint in the literature. It is a single life-time budget

constraint of the consumer where prices have been eliminated using the first order conditions of his maximization problem. Constraint (9) states that the nominal interest rate is greater than 1, since for the consumer's problem a first order condition is that the marginal rate of substitution between informal cash good and informal credit good is equal to the nominal interest rate. (10) assures that informal credit good is not taxed, and constraint (11) imposes the requirement that the consumption tax in the formal sector be the same for both cash and credit goods. Here, it should be noted that when the government increases inflation to reduce the distortion between informal cash and formal credit goods, it increases the distortion between formal cash and formal credit goods at the same time. Thus there is a trade-off between inflation and consumption taxes. Constraint (12) guarantees that the consumption tax rate is positive; otherwise tax collection costs would be meaningless. And finally, (13) assures that consumption and labor are non-negative.

Hence, the Ramsey Problem is to maximize (1) subject to (7)-(13). Note that conditions (12) and (13) will not be binding in a Ramsey equilibrium.

4 Quantitative Results

As in Nicolini (1998), I assume that the consumer's utility function is such that

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma} \qquad \text{for } c > 0$$

and

$$V(n) = n \qquad \text{for } n \geq 0$$

Let government expenditure, g, be constant for each period, and let us consider the steady state solution to the Ramsey Problem. A period is assumed to be one year. I choose the parameters so that the resulting competitive equilibrium matches US macroeconomic data. Five observations are used in the calibration:

The size of the informal sector relative to the size of the formal economy can be stated as

$$i = \frac{a * y^I(\tau, R) + b * x^I(\tau, R)}{c * x^F(\tau, R) + d * y^F(\tau, R) + g + \tau * (1-\kappa)\big(c * x^F(\tau, R) + d * y^F(\tau, R)\big)}$$

where $y^I(\tau, R)$, $x^I(\tau, R)$, $x^F(\tau, R)$, and $y^F(\tau, R)$ are consumer demand functions[9] given the values of τ, R and σ. Note that the denominator consists of private consumption of the goods traded in

[9]For the utility function specified above, the demand functions are very easy to derive.

the formal sector, government expenditures and the resources wasted during tax collection. For 1989-1993 Schneider and Enste (2000) estimate the informal sector in the US to be between 6.7% and 13.9%. In my calibration of this model I use the midpoint of this range, which is 10.3%.

The size of the government as a fraction of the formal economy in the model is

$$v = \frac{g + \tau * (1 - \kappa) * \left(c * x^F(\tau, R) + d * y^F(\tau, R)\right)}{c * x^F(\tau, R) + d * y^F(\tau, R) + g + (1 - \kappa) * \tau * \left(c * x^F(\tau, R) + d * y^F(\tau, R)\right)}$$

In the US, government consumption was on average 19.75% of official GDP in 1976-2000.

The real quantity of money as a fraction of the official output can be calculated as

$$m = \frac{b * x^I(\tau, R) + c * x^F(\tau, R)}{c * x^F(\tau, R) + d * y^F(\tau, R) + g + (1 - \kappa) * \tau * \left(c * x^F(\tau, R) + d * y^F(\tau, R)\right)}$$

The nominal money supply, M1, was on average 15.08% of the official nominal GDP in 1976-2000 in the US according to the SourceOECD.

As it is explained in the introduction, the informal sector is thought to be more cash-intensive than the formal sector. Thus

$$\frac{b}{a} = \theta * \frac{c}{d}$$

where $\theta \geq 1$ is the relative cash intensity of the informal sector. However, the parameter θ has not been estimated numerically yet for any country. For this reason I first assume that $\theta = 1$ and find a lower bound for the optimal interest rate and an upper bound for the optimal tax rate in the US. Then I assume $\theta = 2$ and $\theta = 5$ and solve the problem again for the cases when the informal sector is twice or five times more cash intensive than the formal sector in order to get an idea of how sensitive the results are to the choice of the parameter θ.

Finally, all four sectors together should total 1. This is the identity equation.

$$a + b + c + d = 1$$

The value of R is obtained from the International Financial Statistics. In the US, the annual net nominal interest rates were 6.67% on average in 1976-2000. The tax rate, τ, is estimated by dividing total net tax revenues by the official output. The tax revenue data is obtained from the SourceOECD website. In the US, net tax revenue was 26.61% of the official GDP in 1976-2000. As in Nicolini (1998), I set $\sigma = 2$ for the US which is consistent with the existing literature. The efficiency parameter of the tax system, κ, is assumed to be 0.9953 based on the OECD estimates.

Table 3 reports the baseline values for the US, and table 4 reports the baseline parameters calibrated using these baseline values. With these calibrated baseline parameters, the Ramsey annual nominal interest rate in the US is estimated at 5.42%. This corresponds to 2.26% annual optimal inflation rate when the annual discount factor, β, is assumed to be 0.97. The optimal tax rate on consumption, on the other hand, is 23.23%; in equilibrium 95.79% of the government expenditures are financed by the tax revenue.

When κ is assumed to be 1, i.e. taxes are not costly to collect, then the optimal annual interest rate is 3.78%, and the optimal annual inflation rate is 0.67%. Thus even when the tax collection cost is small, it changes the government's optimization problem in a crucial way and increases the optimal interest rate significantly.

If the informal sector is assumed to be twice as cash intensive as the formal sector, that is when $\theta = 2$, then the optimal interest rate for the US is estimated to be 7.28%. This estimate is even closer to, albeit higher than, the long-run average interest rate. And if $\theta = 5$, the optimal interest rate is even higher at 10.02%. Unlike most of the previous results in the literature on optimal interest rates, actual and optimal interest rates for the US are quite close to each other. The combination of tax collection costs with the existence of an informal sector appear to explain the deviation from the Friedman Rule in the US in 1976-2000.

I carry out the same analysis for the group of countries in Table 1. The base values for these countries are given in Table 5. For each of these countries, data on interest rates, government expenditures and nominal GDP were obtained from the International Financial Statistics. The average tax rate and the real quantity of money were calculated using the SourceOECD data. The size of the informal sector is taken from Schneider and Enste (2000) and from Öğünç and Yılmaz (2000) — in my estimations I have used the midpoints of the ranges provided in those papers. The efficiency parameter of the tax system, κ, is taken from OECD (2004) with the exceptions of Italy and Mexico. For these two countries no OECD estimates were available, so κ is assumed to be the average value in the sample as 0.99. The preference parameter σ is assumed to be 2 for all these countries, as it was for the US. Again, for each country three values of θ, namely $\theta = 1$, 2, and 5, are considered, and the estimations are performed for each case.

Table 6 reports the optimal interest rates in comparison with 25-year average values and with optimal values reported in Yeşin (2004). One observation is that for many of the countries in the sample the actual and the optimal interest rates are quite close to each other, with the exceptions

of Mexico and Turkey. This is not too surprising considering the commonly held belief that these countries were not on the "correct" side of a seigniorage Laffer curve. Figure 4 shows that Mexico and Turkey are outliers in the sample, even when we assume that cash intensity in the informal sector is 5 times as much as in the formal sector. The disparity between the optimal numbers reported here as opposed to in Yeşin (2004) arises not only due to different models used, but mainly due to different values assumed for tax collection costs[10].

For most of the countries the model predicts optimal interest rates close to the long-run averages and it explains some of the cross-country variation in nominal interest rates. Hence optimality considerations with regard to costly taxes and tax evasion seem to account partly for the differences in monetary policy across countries. Other factors — possibly politico-economic ones — may be responsible for the remaining variation in nominal interest rates. This result is consistent with the findings in Yeşin (2004).

Table 6 and 8 show that the optimal interest rate is increasing and the optimal tax rate is decreasing in the cash intensity of the informal sector, respectively. Intuitively, this makes sense, since the government would rely on the inflation tax more, if the informal sector uses cash much more than the formal sector. However, one should also note that the relative positions of the countries change as the cash intensity of the informal sector increases, e.g. see how much the optimal interest rate increases in the US and in Korea, respectively, when θ increases from 1 to 5 in Figure 4. The fact that optimal interest rates in the US increase much rapidly than in Korea is largely due to the fact that the government spending that needs to be financed in the US is much larger than in Korea. Such a government would have to rely more on seigniorage[11]. This observation can be also seen in Table 7 and Figure 5.

According to the model, all the countries except Mexico and Turkey had lower seigniorage revenues than what was optimal. Table 7 and Figure 6 demonstrate this finding. Comparison of average and optimal tax rates in Table 8 yields to the conclusion that in the sample tax burden was much higher than needed for optimal resource allocation. However, one should note that this is a representative agent model and does not consider the income redistribution purposes of tax

[10]Since the OECD (2004) estimates for tax collection costs have become available only recently, Yeşin (2004) tries to estimate them for the first time for a variety of countries in the literature and does not have accurate numbers in some cases.

[11]This finding is consistent with Nicolini (1998).

collection such as in Denmark or Norway.

Figure 7 exhibits optimal combinations of tax and interest rates for each country in the sample as the cash intensity of the informal sector increases. Note that for some countries, such as Denmark, the sensitivity of optimal tax rate to the parameter θ is relatively higher compared to other countries, such as Turkey, even though the sensitivity of optimal interest rate to the parameter θ seems to be similar in both countries.

Finally, Table 9 lists what fraction of the government spending should be financed by taxes for each country. The rest is paid by new money printing and borrowing. Note that the reliance on tax revenue gets smaller as θ increases. Assuming that the informal sector is more cash intensive makes tax collection less attractive as a way of financing government expenditures.

5 Conclusion

In this paper I consider two potentially important reasons to rely at least partly on inflationary finance: tax collection costs and tax evasion. The model described in Nicolini (1998) is extended to incorporate a costly tax collection system whereas fiat money can be printed without cost. This assumption is crucial, since it changes the government's problem and its optimal decision in a significant way. Although tax collection costs do not appear to be high across countries in the data, they do contribute to the optimal interest rates considerably.

Informal sector is assumed to be in the final goods sector, therefore no explicit assumption is needed regarding the elasticity of substitution between informal and formal labor. Using this simple cash-credit model I compute the optimal interest and tax rates for 11 OECD countries. I find that for many countries in the sample, except Mexico and Turkey, the long-run average interest rates and optimal values implied by the model are quite close to each other, and the variation in monetary policy across countries and the deviations from the Friedman Rule can partly be explained by tax collection costs and tax evasion. Thus the results in Yeşin (2004) are robust to the choice of income or consumption taxes.

I also estimate optimal consumption tax rates and compare them with long-run average tax rates. In all cases, optimal tax rates are found to be smaller than actual rates. Furthermore, the reliance on tax revenue gets smaller as the cash-intensity of the informal sector increases, since inflation tax becomes a more efficient way of collecting revenue. The size of the government turns

out to be an important parameter, as the optimal tax and interest rates depend strongly on the government spending level that needs to be financed. Another contribution of this paper is to use the recently available data set in OECD (2004) on tax collection costs making the estimations more precise.

6 Appendix

If the Consumption Tax on the Formal Cash Good must be Paid in Cash:

If the cash-in-advance constraint (4') is used instead of (4) in the model, Proposition 1 still holds, except that constraint (11) has to be replaced by

$$\frac{U'\big(x_t^F\big)}{U'\big(y_t^F\big)} = \frac{U'\big(x_t^I\big)}{U'\big(y_t^I\big)} \tag{11'}$$

In the calibration, only the formula for money holdings as a fraction of the nominal output changes — since the tax on the formal cash good is assumed to be paid in cash. Hence

$$m = \frac{b * x^I(\tau, R) + (1+\tau) * c * x^F(\tau, R)}{c * x^F(\tau, R) + d * y^F(\tau, R) + g + \tau * (1-\kappa)\big(c * x^F(\tau, R) + d * y^F(\tau, R)\big)}$$

Table 10 lists the optimal interest rates based on the alternative cash-in-advance constraint. The optimal interest rates are not too different from the ones reported in Table 6. However, for low values of the relative cash intensity parameter, i.e. $\theta = 1$ and $\theta = 2$, the optimal interest rates with the alternative specification of the cash-in-advance constraint are lower than before. This is intuitive, since the government would choose to use the inflation tax less if more cash were needed in the formal sector due to consumption taxes on the cash good. However, this difference gets smaller as θ increases. As the informal sector uses cash more intensively, the welfare gain of the formal sector from lower inflation tax becomes less important than the inflation 'tax revenue' loss of the government. Hence for $\theta = 5$ the optimal interest rates based on the two cash-in-advance constraints are much closer to each other. Thus the conclusions in the main text apply here as well.

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Table 1: Tax Collection Costs

Country	Administrative Costs/ Net Revenue Collections (%)			
	2000	2001	2002	average
Australia	1.11	1.27	1.19	1.19
Canada	1.07	1.08	1.20	1.12
Denmark	n.a.	n.a.	0.73	0.73
Italy	n.a.	n.a.	n.a.	n.a.
Korea	0.80	0.85	0.85	0.83
Mexico	n.a.	n.a.	n.a.	n.a.
Norway	n.a.	0.56	0.59	0.58
Portugal	1.60	1.61	1.68	1.63
Spain	n.a.	0.81	0.78	0.80
Turkey	1.94	2.12	0.86	1.64
US	0.43	0.46	0.52	0.47

Source: OECD (2004) estimates based on country survey responses and annual reports of revenue bodies.

Table 2: Net Nominal Interest Rates, Seigniorage Revenues, the Size of the Government and the Size of the Informal Sector (1976-2000)

Country	Interest Rate§ (%)	Seigniorage⋆ (% of GDP)	Government Size† (% of GDP)	Informal Sector‡ (% of GDP)	Tax Revenue∗ (% of GDP)
Australia	11.54	0.36	19.16	10.1-15.3	28.67
Canada	8.86	0.25	19.82	10.0-13.5	35.04
Denmark	7.20	0.62	26.01	9.4-16.9	46.95
Italy	12.22	0.59	18.17	19.6-24.0	36.68
Korea	7.56	0.92	10.53	20.3-38.0	19.06
Mexico	30.80	2.90	10.09	27.1-49.0	17.62
Norway	9.11	0.38	20.06	5.9-16.7	41.78
Portugal	14.69	0.85	16.49	15.6-16.8	28.45
Spain	10.80	0.96	14.90	16.1-22.9	29.43
Turkey	43.17	3.14	11.46	15.7-46.2	21.92
U.S.	6.67	0.39	19.75	6.7-13.9	26.61

Source:
§ : 25-year averages of end-of-period discount rates, series 60 of the International Financial Statistics.
⋆ : 25-year averages of change in reserve money divided by nominal GDP, series 14 and 99b of the International Financial Statistics, respectively.
† : Government sizes are computed as the ratio of government consumption expenditures to domestic output, series 91f and 99b of the International Financial Statistics, respectively. They are averages over the period 1976-2000.
‡ : Informal sector size estimates are taken from Schneider and Enste (2000) except for Turkey which is taken from Öğünç and Yılmaz (2000). The numbers are ratios of output produced by the informal sector to output produced by the formal sector in a year, for the period 1989-1993. The output of the informal sector can be estimated using a variety of methods: Surveys and tax audits are examples of the direct approach; currency demand method, physical input (electricity consumption) method, and the Multiple Indicators Multiple Causes model are examples of the indirect approach. A range implies that more than one method was used for that country.
∗ : Tax revenue data is taken from SourceOECD, and GDP from International Financial Statistics.

Table 3: Baseline Values

	R	τ	σ	κ	i	v	m	θ
US	1.0667	0.2661	2	0.9953	0.1030	0.1975	0.1508	1

Table 4: Baseline Parameters

	a	b	c	d	g
US	0.0850	0.0174	0.1528	0.7447	0.1945

Table 5: Base Values for the Other Countries

	R	τ	σ	κ	i	v	m
Australia	1.1154	0.2867	2	0.9881	0.1270	0.1916	0.1366
Canada	1.0886	0.3504	2	0.9888	0.1175	0.1982	0.0776
Denmark	1.0720	0.4695	2	0.9927	0.1315	0.2601	0.3168
Italy	1.1222	0.3668	2	0.9900	0.2180	0.1817	0.3903
Korea	1.0756	0.1906	2	0.9917	0.2915	0.1053	0.2819
Mexico	1.3080	0.1762	2	0.9900	0.3805	0.1009	0.0925
Norway	1.0911	0.4178	2	0.9943	0.1130	0.2006	0.2439
Portugal	1.1469	0.2845	2	0.9837	0.1620	0.1649	0.2712
Spain	1.1080	0.2943	2	0.9921	0.1950	0.1490	0.2573
Turkey	1.4317	0.2192	2	0.9836	0.3095	0.1146	0.0725

Table 6: Annual Net Nominal Interest Rates (%) — Actual and Ramsey Values: Varying Relative Cash-intensity of the Informal Sector

Country	Actual Interest Rate average (1976-2000)	Optimal when $\theta = 1$	Optimal when $\theta = 2$	Optimal when $\theta = 5$	Yeşin (2004)
Australia	11.54	8.08	9.97	12.77	11.31
Canada	8.86	7.31	9.42	12.92	10.83
Denmark	7.20	11.20	13.00	14.94	19.38
Italy	12.22	9.79	10.74	11.75	18.13
Korea	7.56	5.75	6.61	7.62	11.67
Mexico	30.80	6.02	7.40	9.16	14.27
Norway	9.11	6.19	7.66	9.52	9.22
Portugal	14.69	9.82	10.86	12.13	11.83
Spain	10.80	6.50	7.72	9.23	11.46
Turkey	43.17	7.77	9.29	11.36	20.06
US	6.67	5.42	7.28	10.02	8.12

Source: International Financial Statistics, the author's estimations based on the model, and optimal interest rates reported in Yeşin (2004).

Table 7: Seigniorage Revenue

	% of GDP				% of Gov. Exp.			
Country	Actual	Optimal			Actual	Optimal		
	(1976-2000)	$\theta = 1$	$\theta = 2$	$\theta = 5$	(1976-2000)	$\theta = 1$	$\theta = 2$	$\theta = 5$
Australia	0.36	0.64	0.86	1.16	1.61	3.42	4.60	6.23
Canada	0.25	0.31	0.45	0.66	1.08	1.62	2.35	3.47
Denmark	0.62	2.30	2.74	3.18	2.04	9.27	11.06	12.87
Italy	0.59	2.39	2.68	2.97	1.73	14.19	15.94	17.70
Korea	0.92	0.71	0.92	1.17	6.26	7.04	9.17	11.59
Mexico	2.90	0.72	0.40	0.55	11.48	3.84	4.13	5.66
Norway	0.38	0.91	1.04	1.42	1.09	4.87	5.54	7.58
Portugal	0.85	1.68	1.91	2.18	2.18	10.81	12.28	14.01
Spain	0.96	0.83	1.11	1.43	4.84	5.92	7.89	10.20
Turkey	3.14	0.35	0.59	0.94	15.18	3.26	3.01	4.80
U.S.	0.39	0.33	0.46	0.59	1.26	1.71	4.21	5.45

Source: International Financial Statistics and the author's estimations based on the model.

Table 8: Tax Rates (%)

	Average†	Optimal		
Country	(1976-2000)	$\theta = 1$	$\theta = 2$	$\theta = 5$
Australia	28.67	21.55	21.24	20.78
Canada	35.04	22.74	22.54	22.21
Denmark	46.95	28.25	27.50	26.71
Italy	36.68	15.62	15.20	14.77
Korea	19.06	9.39	9.13	8.83
Mexico	17.62	10.04	9.91	9.74
Norway	41.78	21.18	20.75	20.21
Portugal	28.45	15.25	14.93	14.54
Spain	29.43	14.34	13.99	13.56
Turkey	21.92	11.42	11.30	11.14
U.S.	26.61	23.23	22.88	22.37

Source: SourceOECD and the author's estimations based on the model.

†: Average tax rate for each country was approximated by Net Tax Revenue/GDP.

Table 9: How much Tax Revenue is Optimal to Finance Government Spending

	Optimal†		
Country	$\theta = 1$	$\theta = 2$	$\theta = 5$
Australia	94.00	92.67	90.73
Canada	96.99	96.16	94.84
Denmark	85.75	83.60	81.35
Italy	77.23	75.21	73.15
Korea	83.90	81.63	79.01
Mexico	93.76	92.38	90.68
Norway	91.84	89.99	87.71
Portugal	82.67	80.96	78.91
Spain	87.96	85.82	83.24
Turkey	94.17	93.10	91.67
U.S.	95.79	94.38	92.36

Source: SourceOECD and the author's estimations based on the model.

†: Gross tax revenue as a percentage of government total expenditures in the Ramsey equilibrium.

Table 10: Optimal Annual Net Nominal Interest Rates Using the Alternative Cash-in-Advance Constraint (%)

Country	Actual Interest Rate average (1976-2000)	Optimal when $\theta = 1$	Optimal when $\theta = 2$	Optimal when $\theta = 5$
Australia	11.54	6.25	7.96	10.83
Canada	8.86	5.59	7.38	10.74
Denmark	7.20	7.44	9.46	12.10
Italy	12.22	7.82	9.16	10.71
Korea	7.56	5.14	6.10	7.29
Mexico	30.80	5.43	6.85	8.75
Norway	9.11	4.47	5.88	7.98
Portugal	14.69	8.09	9.31	10.94
Spain	10.80	5.34	6.65	8.43
Turkey	43.17	6.94	8.48	10.73
US	6.67	3.93	5.51	8.15

Source: International Financial Statistics and the author's estimations based on the model.

Figure 1: Tax Collection Costs and Average Interest Rates

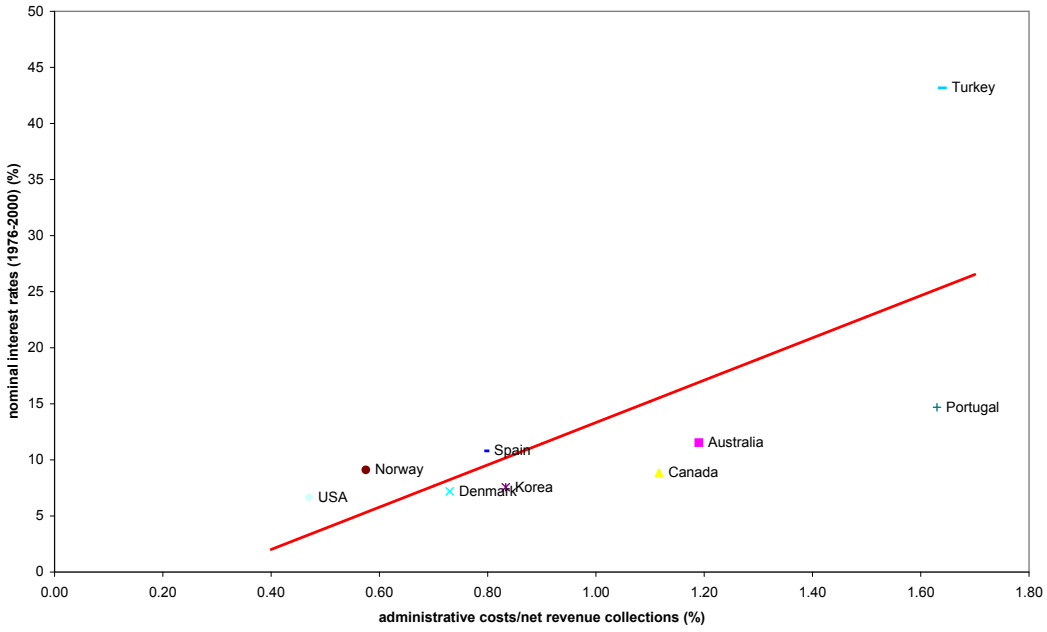


Figure 2: Tax Collection Costs and Average Seigniorage Revenues

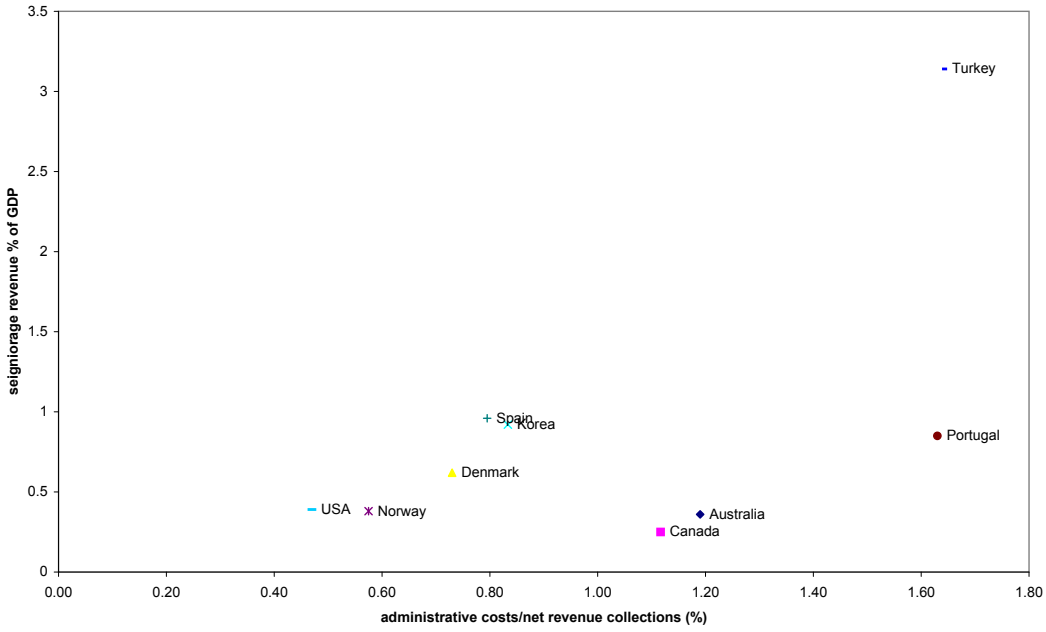


Figure 3: Size of the Informal Sector and Average Interest Rates

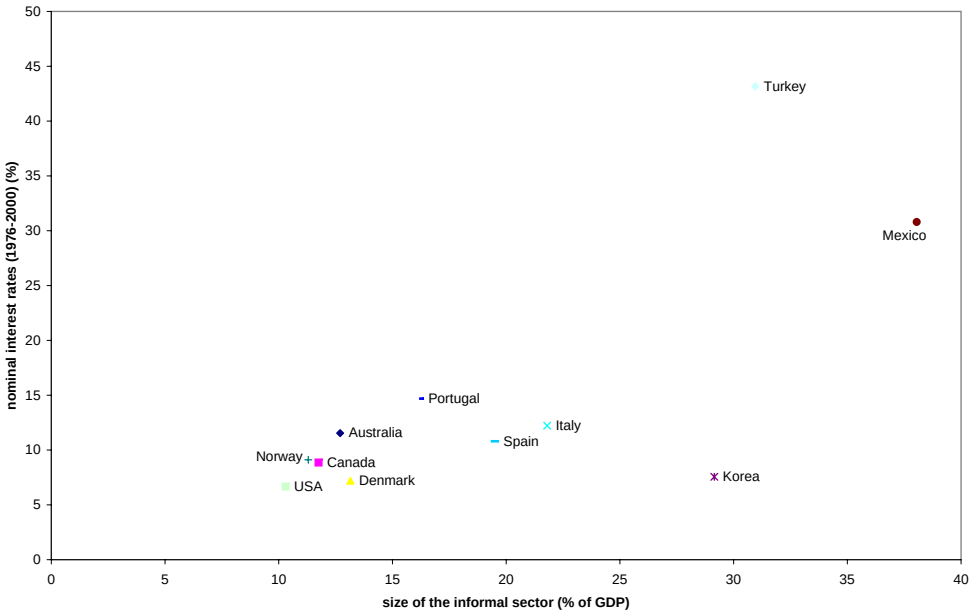


Figure 4: Actual and Optimal Interest Rates

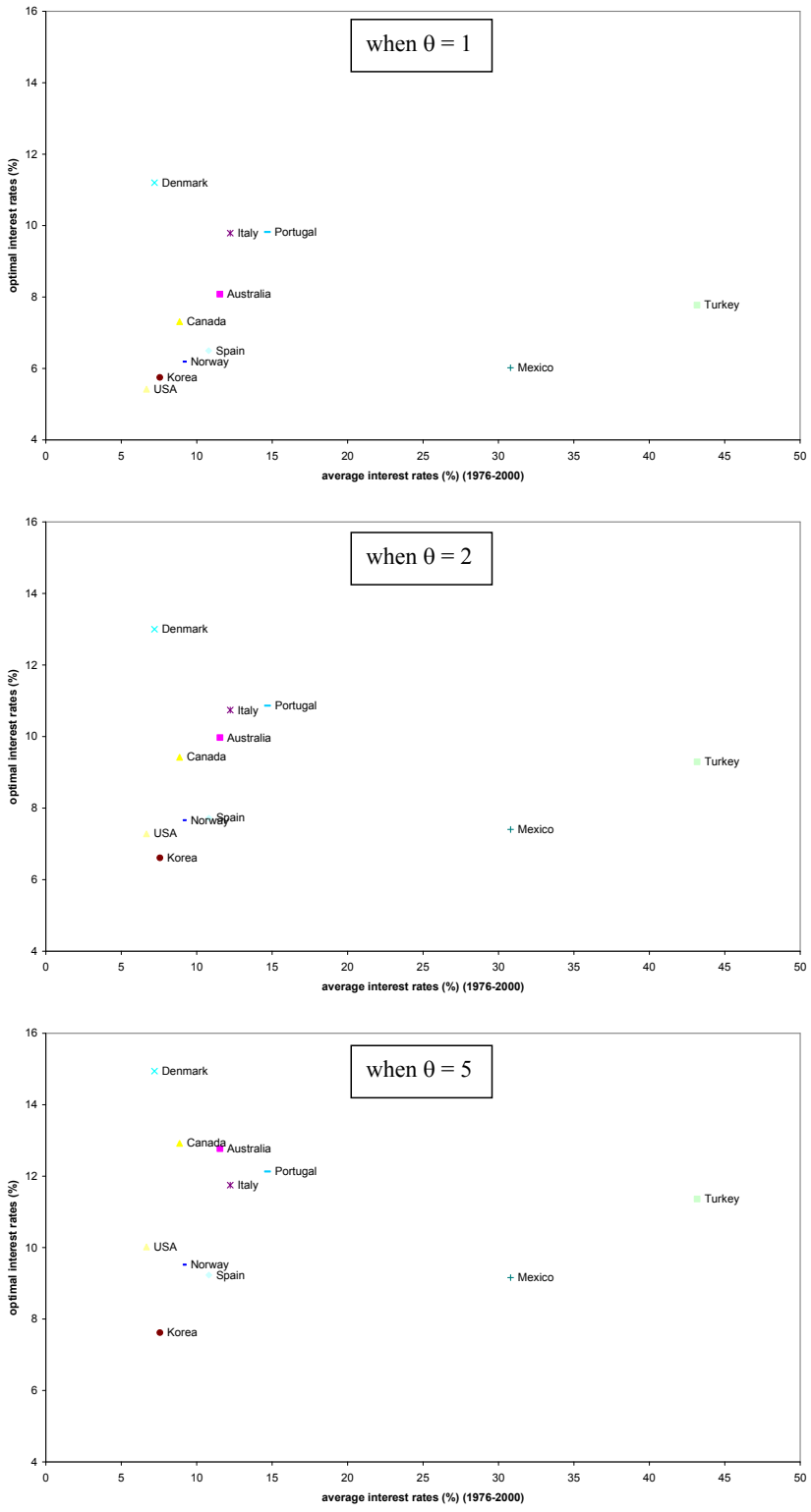


Figure 5: Efficiency of the Tax Collection System and Optimal Interest Rates

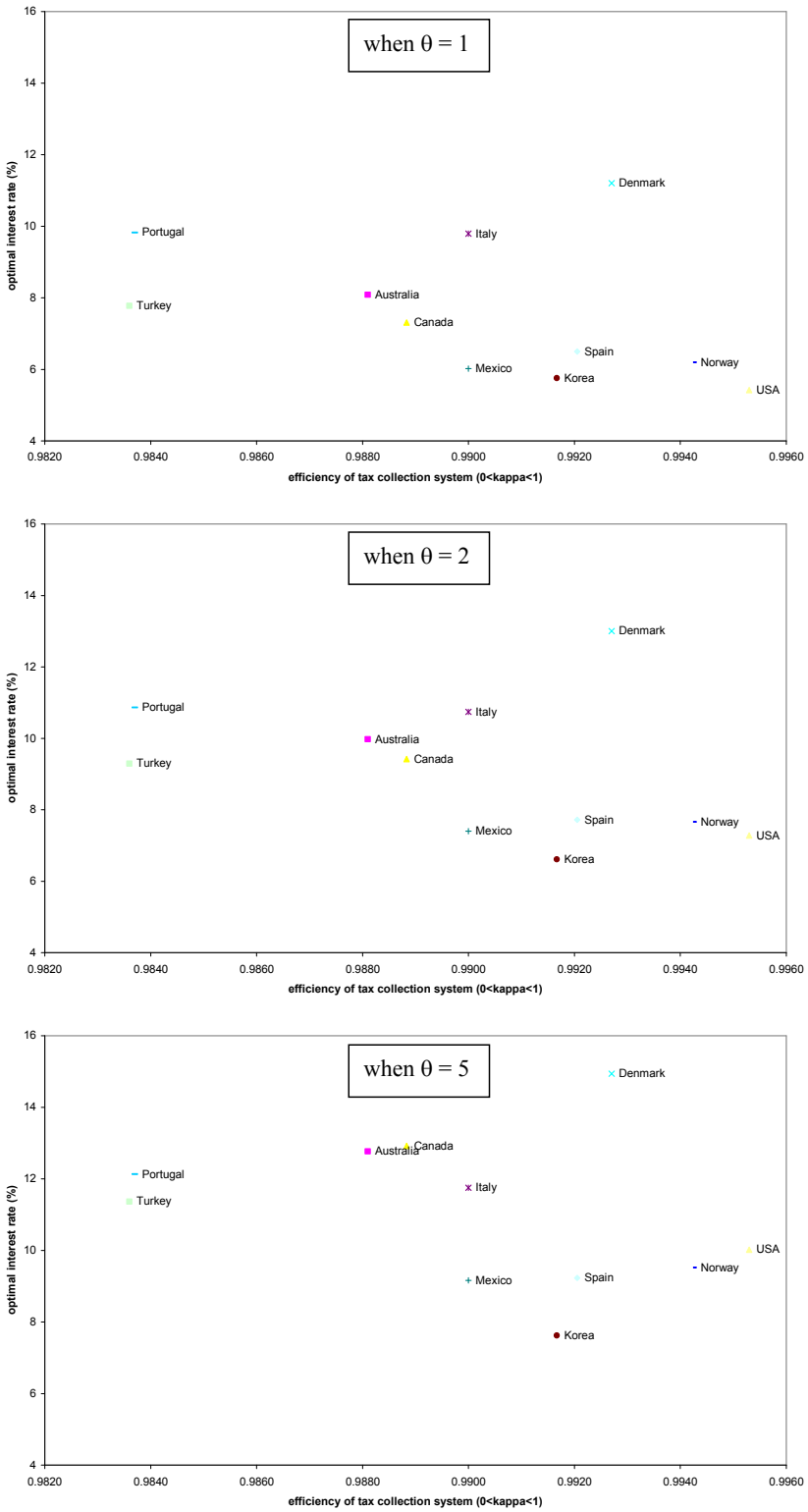
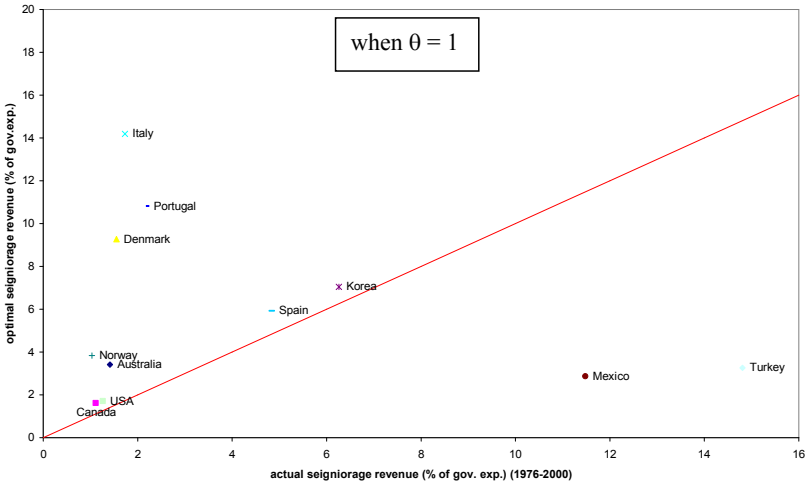
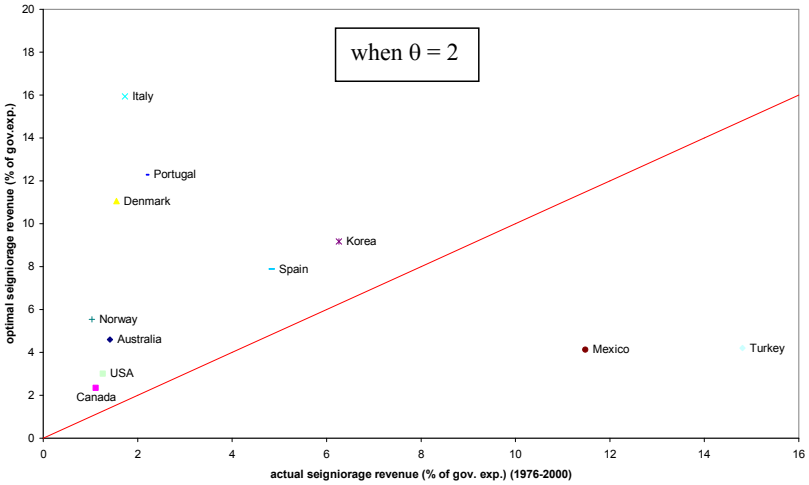


Figure 6: How much Seigniorage Revenue to Finance Government Expenditures is Really Optimal?





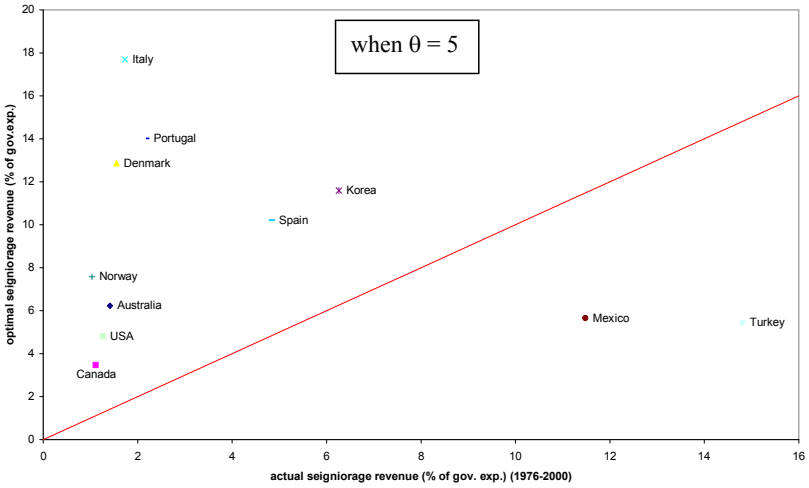


Figure 7: Optimal Tax and Interest Rate Combinations when θ=1, 2, and 5